



Scaling up the Detail in Particle Collisions. Factorization and Resummation for Predictions of Multi-Differential Cross Sections

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Summary

This thesis represents research that has been conducted in the field of elementary particle physics. Although this is a fundamental science and has little to do with the everyday life, it does address questions that appeal to the imagination of many, such as “What is everything made of?”, or “What keeps everything together?”. Exactly because everybody wonders about these things from time to time, this summary is aimed at a broad audience.

The goal of this summary is then to provide the reader with a small glimpse into the world of elementary particles and the modest contribution that this thesis makes to it.

The Standard Model

Although the idea that all matter is built up from tiny building blocks is thousands of years old, it was not until the nineteenth century that the first experimental evidence for this was provided. Discoveries followed one another in rapid succession and at the moment over a hundred different building blocks are known. These particles are called *atoms* and are about a million times as small as the width of a human hair.

With the discovery of the atom, the search for the most fundamental building blocks of nature seemed to have come to an end. It was therefore a big surprise when experiments conducted around the twentieth century showed that the atoms themselves consisted of even smaller particles. Every atom was found to contain a positively charged nucleus, surrounded by a cloud of negatively charged particles, called *electrons*, each with an electrical charge of -1 . The atomic nucleus is in turn made up out of *protons* and *neutrons*. The proton has an electrical charge of $+1$, exactly opposite to the charge of the electron. Neutrons are, as their name suggests, neutral and do not carry any electrical charge. Since atoms as a whole are electrically neutral, they must contain an equal amount of protons and electrons. The exact number of protons (and electrons) in an atom determines the specific type of atom. For example, a helium atom contains two protons and two electrons, while an iron atom consists of twenty-six protons and electrons. The number of neutrons in an atom may vary, corresponding to different so-called isotopes.

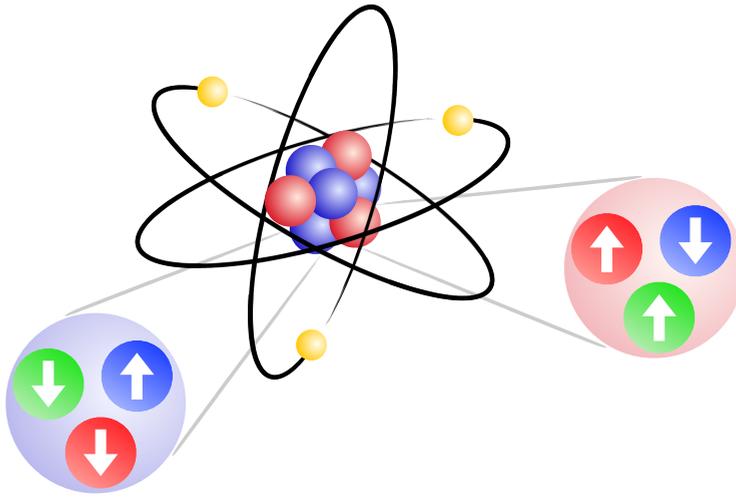


Figure 1 A schematic representation of an atom. Electrons (the yellow orbs) orbit around the nucleus, which contains protons (the red orbs) and neutrons (the blue orbs). The up- and down-quarks (the orbs with arrows pointing up or down) are visible in the enlarged pictures of the proton and neutron.

Even though the existence of only three fundamental building blocks seemed very elegant, protons and neutrons were also found to consist of smaller particles. These particles are called *up-quarks* and *down-quarks* and have an electrical charge of $+2/3$ and $-1/3$ respectively. A proton contains two up-quarks and one down-quark, while a neutron carries two down-quarks and a single up-quark. A graphical representation of an atom can be found in figure 1. Apart from electrical charge, the quarks turn out to carry an additional type of charge: *color charge*. Despite the somewhat confusing name, this charge has nothing to do with actual color. Where electrical charge comes in two sorts (positive and negative), there are three possible color charges, denoted by *red*, *green* and *blue*. Just as the combination of a positively charged particle and a negatively charged particle is electrically neutral, three particles with a red, green and blue color charge together form a color-neutral combination (also known as *white*). The three quarks inside a proton (or inside a neutron) all have a different color, so protons (and neutrons) carry no net color charge. Only color-neutral particles such as protons and neutrons can be observed directly.

Up-quarks, down-quarks and electrons are currently believed to be fundamental particles of matter: they cannot be divided into smaller particles. There is one additional matter particle, called the *electron-neutrino*, which is not im-

portant here, but is mentioned merely for completeness. All known matter is eventually built from these four elementary particles.

The existence of the four matter particles is predicted by a theory developed around 1967 that carries the somewhat pretentious name *the Standard Model*. The Standard Model additionally predicts two exact copies of each of the four matter particles, with the only difference that these copies have a larger mass. Each of these three collections, all containing four matter particles, is called a *generation*.

Apart from the existence of matter particles, the Standard Model also describes the forces that act between these particles. There are three forces within the Standard Model: *electromagnetism*, the *strong nuclear force* and the *weak nuclear force*. The fourth and final known force, gravity, is not described by the Standard Model. Particles are only affected by a certain force if they carry a specific type of charge. For example, particles are only affected by the electromagnetic force if they are electrically charged and only by the strong nuclear force if they carry a color charge.

The forces between the matter particles are transmitted by force-carrying particles. The force carrier of the electromagnetic force is the *photon*, the particle that light is also made of. The strong nuclear force is transferred by the *gluon*. The three quarks in the proton are thus held together because they exchange gluons and in doing so exert an attractive force upon each other. The strong nuclear force has a fitting name: the force between two quarks in the proton is roughly equal to the force required to lift three male African elephants. The crucial difference between photons and gluons is the fact that photons themselves are not electrically charged, but gluons do carry a color charge. This then means that gluons also affect one another through the strong nuclear force.

The third force, the weak nuclear force, is transmitted through so-called *W*- and *Z*-*particles* and is responsible for the radioactive decay of some atoms. The final particle that is contained within the Standard Model is the *Higgs particle*. The Higgs particle is not a matter particle or force carrier, but provides a mechanism through which the other particles acquire their respective masses.

A schematic overview of the particle content of the Standard Model is shown in figure 2. Every particle whose existence the Standard Model predicts has been discovered in experiments. Furthermore, no elementary particle that is not predicted by the Standard Model has ever been found. Hence, the Standard Model is an enormously successful theory. Despite its successes though, it

Matter particles			Force carriers	
I	II	III		
1968 <i>u</i> up-quark	1974 <i>c</i> charm-quark	1995 <i>t</i> top-quark	1979 <i>g</i> gluon	
1968 <i>d</i> down-quark	1968 <i>s</i> strange-quark	1977 <i>b</i> bottom-quark	1923 γ photon	
1897 <i>e</i> electron	1936 μ muon	1975 τ tau	1983 <i>W</i> W-particle	
1956 ν_e electron-neutrino	1962 ν_μ muon-neutrino	2000 ν_τ tau-neutrino	1983 <i>Z</i> Z-particle	2012 <i>H</i> Higgs particle

Figure 2 The particles that are described by the Standard Model and the year of their discovery. The first three columns represent the three generations of matter particles, while the fourth column contains the force carries.

cannot be the ultimate theory of the universe. One of the many both theoretical and experimental motivations for this is the fact that gravity is not contained within the Standard Model.

The search for new particles that are not predicted by the Standard Model is one of the biggest challenges of modern-day particle physics.

Searching for particles

The method that is used to search for new particles is based on what is probably the most famous equation in physics:

$$E = mc^2. \quad (1)$$

Here E stands for energy, m for mass and c is a constant¹. What this formula then entails, is that energy and mass can be converted into one another according to some exchange rate, which happens to be c^2 . The conversion of

¹To be precise, it is the speed of light, roughly equal to a billion kilometers per hour.

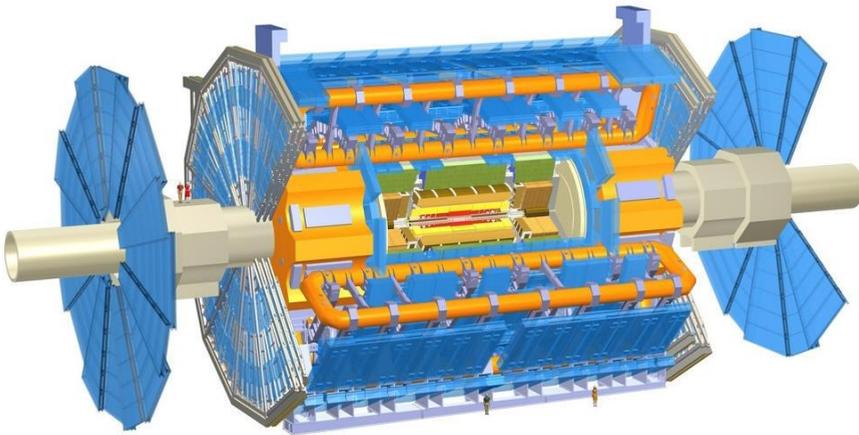


Figure 3 The ATLAS detector. The detector is 46 meters long and has a diameter of 25 meters. The protons enter the detector from both sides and collide in the middle. Around this location, multiple layers of advanced measuring equipment are installed, allowing for the detection of various kinds of particles. The displayed people show the scale of the detector.

mass into energy occurs for example in the generation of nuclear energy, or in the detonation of an atomic bomb. The exact same exchange rate applies in the other direction as well, so energy can also be converted into mass.

The current experimental setup that uses this principle is based in Geneva and is called the Large Hadron Collider (LHC). The LHC is a 27 kilometer long, circular tunnel, built about a hundred meters under the surface. In this tunnel, protons are accelerated in opposite directions until they revolve around the tunnel more than 11 000 times per second. The protons are then made to collide, allowing the quarks (and gluons) inside the protons to interact with each other. Because of the enormous amount of energy that is stored in the motion of the protons, new particles may be created in these interactions through equation (1). These might be particles from the Standard Model that are already known, but may also be new, as of yet undiscovered particles.

Many detectors are set up near the location where the protons collide (see for example figure 3), in order to be able to measure the end products of the interactions that take place. There is, however, a catch: most particles decay into different particles before reaching the detectors. From a single measured final state it is therefore impossible to determine whether a particle from the Standard Model, or a new particle was created in the original interaction.

However, by performing a great number of collisions, at the LHC about forty million per second, and counting the amount of times that a certain final state is measured, the probability of that final state to occur can be determined. This measured probability can then be compared against the prediction from the Standard Model. If the experimentally determined probability of a certain final state is found to be larger than predicted by the Standard Model, it may be concluded that particles must have been created that lead to that particular final state, but are not included in the Standard Model.

Theoretical predictions

Through the use of the Standard Model, the probability for a specific process to occur can be calculated. By adding the probabilities of all processes that lead to a certain final state, the probability of that final state can be determined. This may be compared with rolling two dice, where the final state is the total number of pips that the dice show. By throwing the two dice many times and counting how often some final state, for example ‘ten pips’, occurs, the probability of that final state can be determined experimentally.

To theoretically predict this probability, the probabilities of all possibilities that lead to this final state have to be added. There are three possibilities that lead to ‘ten pips’, namely: , , and . The total amount of possible outcomes when throwing two dice is equal to $6 \times 6 = 36$, for each of the six possibilities of the first die, the second die can take six different values. For the case of fair dice, the probability of each possibility is the same, so the total probability of the final state ‘ten pips’ is equal to $3/36$, which comes down to a little over eight percent.

Individual processes in particles physics are often represented graphically by so-called *Feynman diagrams*. An example of a Feynman diagram is shown on the left-hand side in figure 4, where the particles A and B on the left represent the initial state (the particles that collide) and the particles C and D on the right the final state. By drawing and calculating all possible Feynman diagrams that give rise to the same final state (the particles C and D), the total probability of that state can be predicted.

There is, however, a complication: particles can temporarily split into two different particles, which may subsequently recombine into a single particle. This is for example the case in the Feynman diagram on the right in figure 4, which has the same final state as the left diagram. Since a particle resulting from such a splitting may itself split again, this gives rise to an infinite amount of possibilities that all lead to the same final state. As calculating infinitely

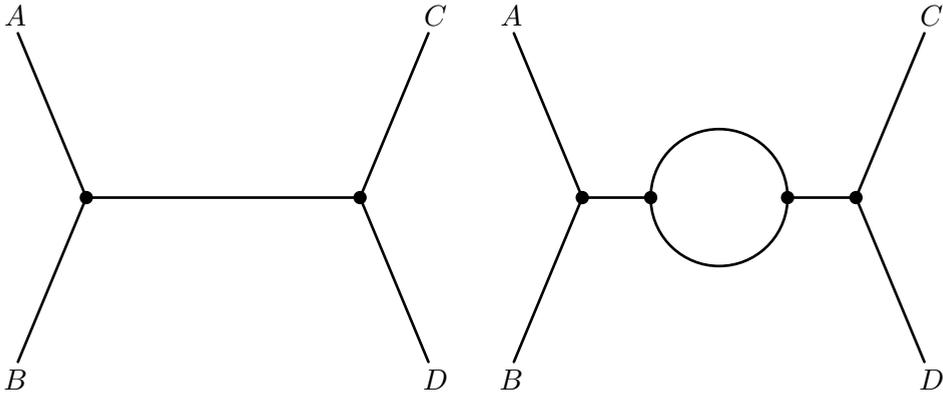


Figure 4 Two Feynman diagrams with possible scenarios in which the incoming particles A and B give rise to the final state composed of the particles C and D .

many diagrams would take a lot of time, this seems to be a problem.

The solution to this issue follows from the fact that every interaction between two particles (every black dot in the diagrams in figure 4) reduces the probability of a diagram. The exact factor by which the probability is reduced depends on the type of interactions. For example, for interactions between quarks and gluons, which are governed by the strong nuclear force, the probability of a certain scenario decreases roughly by a factor of ten for every two interactions. For these strong interactions, this reduction factor, known as a *coupling constant*, is denoted by the symbol α_s .

The infinite amount of scenarios that lead to a specific final state can then be organized according to the amount of interactions (black dots) that appear per scenario as

$$\text{Probability of final state} = c_0 + \alpha_s \times c_1 + \alpha_s^2 \times c_2 + \dots, \quad (2)$$

where the dots denote the fact that the series continues to infinity. The first term on the right-hand side, c_0 , represents the probability that no interaction takes place. In that case, the final state has to be equal to the initial state. The next term, $\alpha_s \times c_1$, is the combined probability of all the diagrams that contain exactly two interactions (like the left diagram in figure 4). Because of the reduction factor α_s , this term is about ten times as small as the previous term. The third term, $\alpha_s^2 \times c_2$, represents the probability of all diagrams with exactly four interactions (like the diagram on the right in figure 4). This term contains two instances of the reduction factor, hence it is about $10 \times 10 = 100$ times as small as the first term c_0 .

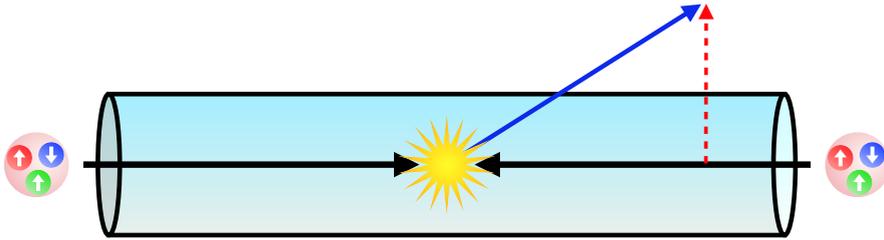


Figure 5 A collision between two incoming protons (the black arrows). The momentum of one of the produced particles is indicated by the blue arrow. The transverse momentum is the degree to which this momentum points in a direction perpendicular to the protons, indicated by the red dashed arrow.

Because every next term in equation (2) is smaller than the previous term, the series may be truncated at a given point, provided that one accepts that the prediction is no longer exact, but is an approximation instead. For example, if only the terms c_0 and $\alpha_s \times c_1$ are taken into account, the error that is made is proportional to the reduction factor of the third term, which amounts to $\alpha_s^2 \approx 1/100$. This shows the power of this method: By calculating only two terms in an infinite series, the true answer may be approximated to 1% accuracy.

Multiple measurements

Apart from the probability that a specific final state occurs, experiments are also able to measure certain properties of that state. An example of a property that can be measured is the so-called *transverse momentum*. Momentum is the amount of energy with which a particle moves in a certain direction. The term ‘transverse’ indicates that the momentum is measured in the direction perpendicular to the direction in which the protons at the LHC move before they collide, see figure 5.

The meaning of momentum becomes clear in an example from everyday life: There is more energy in a car moving with 50 kilometers per hour than there is in a car moving 30 kilometers per hour. The first car then has a larger momentum than the second. A fully loaded truck that drives at 50 kilometers per hour contains more energy than the car driving at 50 kilometers per hour, so the truck has an even larger momentum.

In nature, (transverse) momentum is a conserved quantity. It cannot be lost, only transferred from one particle to the next. In the scenario depicted in the diagrams in figure 4, the transverse momentum of the final state then has to be equal to the transverse momentum of the initial state. Since the momen-

tum is measured in the direction perpendicular to the incoming protons, the transverse momentum of the initial state (and therefore also the final state) has to be zero by definition. What is then the reason that nonzero transverse momenta are measured at the LHC?

Before the incoming particles are considered as the initial state, so before the diagrams in figure 4 begin, these particles can emit radiation. This radiation consists of so-called *soft* and *collinear* particles. Soft particles carry very little energy, so little in fact that they might not even be measured by the detectors. Collinear particles propagate in exactly the same direction as some other particle, so they are often also not detected as individual particles. By emitting this radiation, the incoming particles can obtain a transverse momentum that, by momentum conservation, must be equal (but opposite) to the transverse momentum of the soft and collinear radiation. The transverse momentum of the final state is then completely determined by the soft and collinear radiation, which is difficult to detect.

All measurements considered in this thesis are affected in one way or another by the soft and collinear radiation.

The energy of the soft and collinear radiation is vastly lower than the energy with which the incoming particles collide and eventually form the measured final state. Combined measurements of the probability of a certain final state and the transverse momentum are thus sensitive to processes that occur at two completely different energies.

Simultaneously considering a process occurring at a small energy E_{small} and a process occurring at a large energy E_{large} , leads to a problem in the theoretical calculation. It turns out that the coefficients c_1, c_2, \dots develop a dependence on the ratio of the energies, $E_{\text{large}}/E_{\text{small}}$. To be more specific, this ratio appears in every coefficient to the same power as the power of the coupling constant α_s that corresponds to that coefficient. The coefficient c_2 , for example, depends on the ratio $(E_{\text{large}}/E_{\text{small}})^2$. A typical value of this ratio might be $E_{\text{large}}/E_{\text{small}} = 10$. In that case, this ratio effectively cancels the suppression from α_s in each term. The result is then that every term in equation (2) is roughly of the same size again, so that the series cannot be truncated anymore.

Effective theories, factorization and resummation

Because the issue occurs due to the attempt to describe two completely different processes simultaneously, the solution is very intuitive: try to separate the two processes and consider them individually.

The measurement of the transverse momentum is fully determined by the soft and collinear radiation and is completely insensitive to the energetic interaction between the two colliding particles. This measurement might then just as well be described by a simplified version of the Standard Model, from which everything that has to do with the energetic collision of the incoming particles is omitted. What remains after all (to this measurement) irrelevant information has been removed, is called an *effective theory*.

The principle of effective theories can be found in everyday life as well. A mason, for example, does not have to take the attractive force between atoms into account in order to build a house.

By means of an effective theory, the series in equation (2) can be split into two different series, one depending only on E_{small} and the other only depending on E_{large} . Such a division is called *factorization* since the series is split into two (independent) factors. As both of these factors now depend on only a single energy, they can be calculated individually, without the appearance of any large ratios. Both series can then be truncated after a certain amount of terms.

These factors are eventually recombined to obtain a single result. This process is known as *resummation* and effectively ensures that the ratios of $E_{\text{large}}/E_{\text{small}}$ no longer appear in the coefficients of the series in equation (2). Instead, the total contribution of all these ratios is taken into account in a single, overarching coefficient that is calculated during the resummation process.

In general, it is not easy to prove that such a factorization is actually possible. Furthermore, the procedure depends on the exact measurement (in this case the transverse momentum) that is being done. For every variable that one would like to measure, a new factorization has to be proven.

The work described in this thesis encompasses the development of factorizations and resummations of processes in which two or more measurement are considered simultaneously. The procedure described above is significantly complicated in these situations. Every measurement might in principle correspond to a different energy, so multiple ratios of energies can occur in the coefficients of the series in (2). In that case, it has to be proven that the series can be factorized into as many factors as there are measurements. On the other hand, some of the measurements might also correspond to the same energies, so that a factorization into a smaller number of factors might be in order. One of the most important challenges when considering multiple measurements simultaneously is then to figure out which factorization has to be used in which situation.

In chapter 4, the simultaneous resummation of two measurements is carried

out, one of which is the transverse momentum. This resummation enables the calculation of predictions that are valid for all possible energies that these measurements might have with respect to each other and to the energy of the incoming, colliding particles. The final predictions are shown in three-dimensional graphs in figure [4.11](#). The results in this chapter represent the first predictions of the combination of the two measurements under consideration. This particular combination of measurements has been considered at experiments and may also be used to improve simulations of particle collisions made by certain computer programs.

The simultaneous factorization and resummation of two measurements is also the subject of chapter [5](#). Again, one of the measurements is the transverse momentum. Although the measurements in this chapter are not factorized and resummed for the first time, the developed procedure does lead to a more accurate result than the previous methods. This framework may be applied to the production of Higgs bosons, or that of hypothetical, unknown, heavy particles that are not predicted by the Standard Model.

The factorization that is derived in chapter [6](#) is a more general version than currently known in the literature and has many future applications. It is the first factorization that describes both the soft and the collinear behavior of a certain class of processes.

Finally, in chapter [7](#) the question of how many measurements can be resummed simultaneously is raised. The idea behind this question is the fact that all measurements ultimately measure some property of the same set of particles (the final state). It might then be the case that every property of the final state is known after a certain amount of measurements, so that subsequent measurements do not provide any new information. This means that additional measurements no longer give rise to new ratios of energies in the coefficients in equation [\(2\)](#). In that case, the resummation of these extra measurements is not required. The results of this chapter imply that the resummation of two measurements provides a large improvement over resumming only a single measurement. A reassuring conclusion, given the subjects of the other chapters in this thesis. In addition, the results show that resumming even more measurements provides an increasingly smaller improvement.

In short, the research described in this thesis is aimed at improving the precision of the predictions that the Standard Model makes. By determining the probability of these known processes with an increasing accuracy, it will hopefully become possible to discover new and unknown particles. After all, to find a needle in a haystack, one first has to know exactly what hay looks like.