Gauge freedom in transport through quantum dots: Interaction-induced geometric pumping

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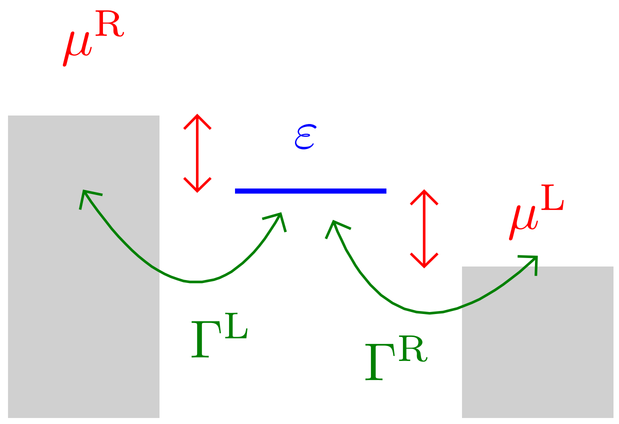
• Adiabatic pumping in open quantum systems



instantaneous current
(frozen parameters)

$$I(t) \;=\; \underset{\uparrow\ \text{instantaneous current (frozen parameters)}}{I^i\,[R(t)]} + \boxed{I^a\big[\,R(t), \dot{R}(t)\,\big]}$$

adiabatic-response / pumping current $\propto \Omega$

Adiabatic periodic driving:

- parameters $R(t)$
- frequency $\Omega = 2\pi / T$

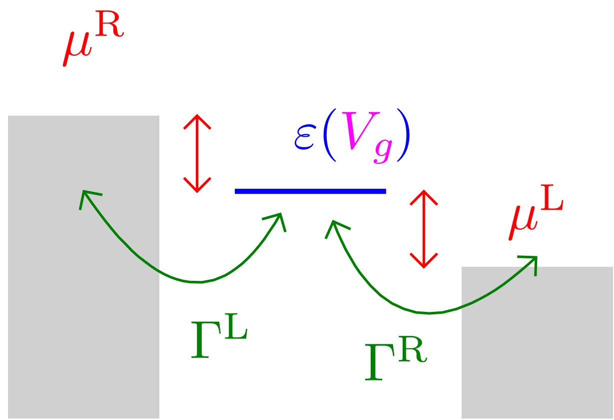
Transferred charge / cycle

$$Q \;=\; \underbrace{\int_0^T \mathrm{d}t\, I^i\,[R(t)]}_{\propto T = \frac{2\pi}{\Omega}} + \underbrace{\boxed{\oint \mathrm{d}R\, \frac{\partial I^a}{\partial \dot{R}(t)}[R(t), 0]}}_{\Omega \text{ independent}}$$

• Pumping charge through a quantum dot

Voltage driving $\boxed{R = (V_g, V)}$

$\Omega \ll \Gamma \ll T,\ V = \mu^{\mathrm{L}} - \mu^{\mathrm{R}} \ll \boxed{U}$

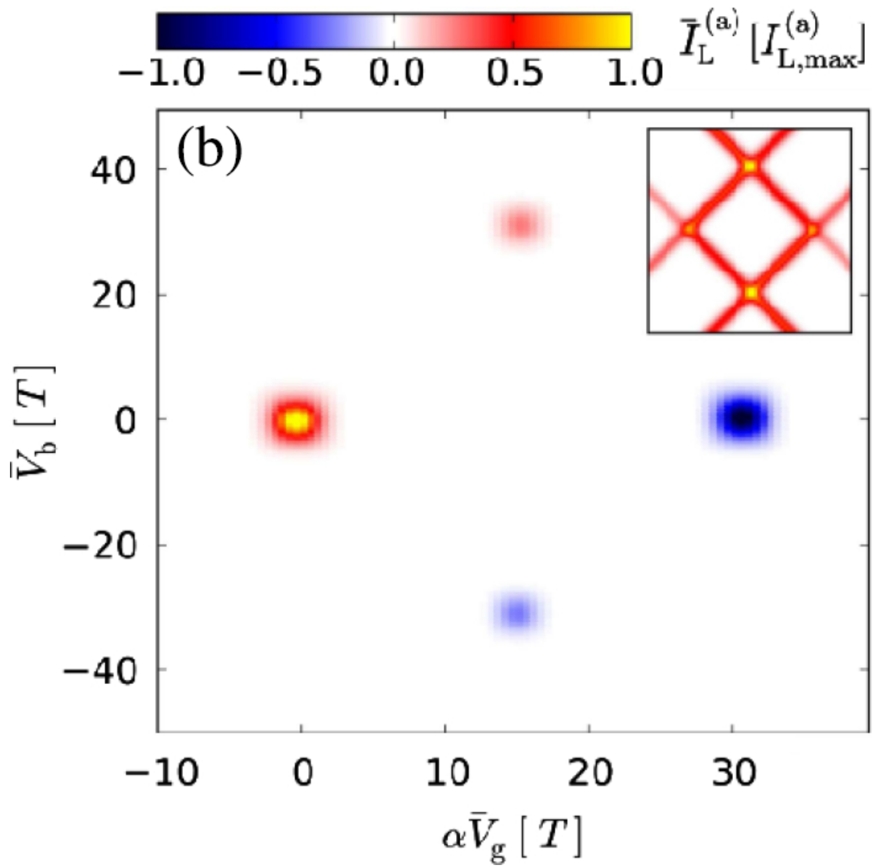


- Detect *changes* in degeneracies
- Determine junction asymmetry
- $\boxed{\text{Interaction } U = 0 \Rightarrow \text{pumping} = 0\,!}$

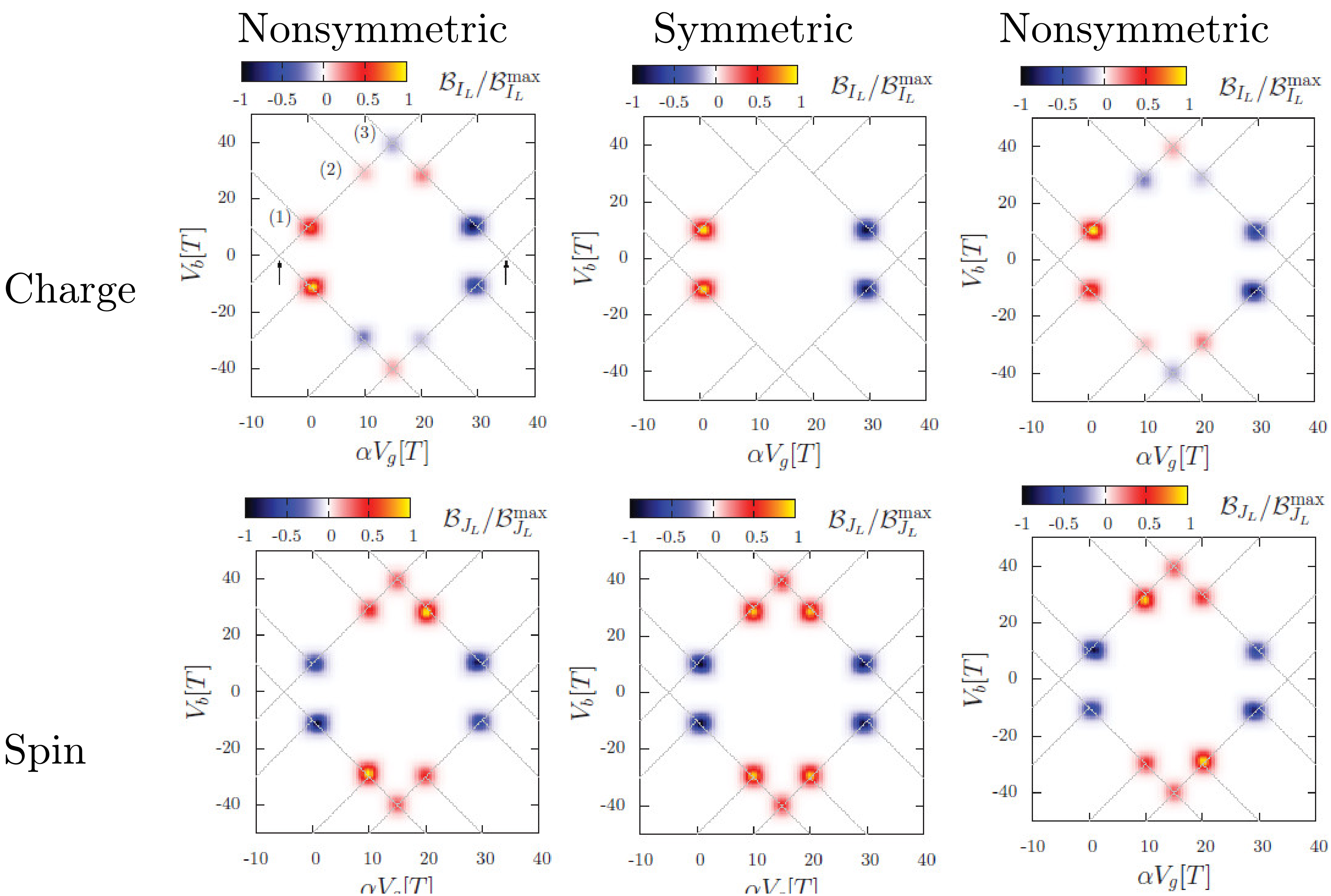
→ chemical reactions [Sinitsyn and Nemenman, 2007]
→ quantum dots [Calvo et al., 2012, Haupt et al., 2013, Yuge et al., 2013, Yoshii and Hayakawa, 2013, Nakajima et al., 2015]
→ heat pumping [Ren et al., 2010]

Pumped charge / cycle

[Reckermann et al., 2010]



• Pumping charge & spin in B-field



→ Also: zero-frequency pumping noise: splitting of "spots", more features [Riwar et al., 2013]

• Geometric phases *due to strong interaction* ?

$$Q^a = \oint \mathrm{d}R\, A[R] = \text{geometric “phase” ?}$$

$$A[R]\dot{R}(t) = I^a\big[R(t), \dot{R}(t)\big] = \text{geometric connection?}$$

Outline:

1. Transport theory: **density operator** + observable ?
2. **Conditions** for pumping: **interaction** required ?
3. **Gauge freedom & geometry** ? → prerequisite for topological pumping

Combination + extension of ideas:

[Sarandy and Lidar, 2005, Sarandy and Lidar, 2006], [Sinitsyn and Nemenman, 2007, Sinitsyn, 2009], [Avron et al., 2012], [Landsberg, 1992, Landsberg, 1993, Andersson, 2003a, Andersson, 2003b]

other approaches → end of talk

• Density-operator transport theory

- **Supervectors**: bra-ket notation (Hilbert-Schmidt / Liouville space)

$$\text{trace} \quad \mathrm{Tr}\,\mathbb{1}\,\bullet \; = \; (\mathbb{1}| \qquad\qquad |\rho(t)) \; = \; \rho(t) \quad \text{density operator}$$

E.g. trace-normalization

$$\mathrm{Tr}\,\rho(t) \; = \; (\mathbb{1}|\rho(t)) \; = \; 1$$

- **Super operators**: map of operators
E.g. Liouvillian

$$L\,|\rho) \; = \; [H\,,\,\rho]_{-}$$

• Density-operator time evolution

$$|\rho(t)) \;=\; \left[\operatorname*{Tr}_{\text{res}} e^{-i\int_{-\infty}^{t} \mathrm{d}\tau L^{\text{tot}}(\tau)} \rho^{\text{res}}\right] |\rho(-\infty)), \qquad L^{\text{tot}}(t) \;=\; L[R(t)] + L^{\text{res}}[R(t)] + L^{\text{tun}}[R(t)]$$

$$\rho(\overset{\downarrow}{t}) \;=\; \text{stationary, time-dependent due to } \overset{\downarrow}{R}(t)$$

Kinetic equation = time-nonlocal quantum master equation

$$\partial_t|\rho(t)) \;=\; -i\,L|\rho(t)) + \int_{-\infty}^{t} \mathrm{d}t'\, W(t,t')\,|\rho(t'))$$

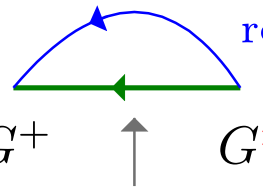
Time-evolution **kernel**: expansion in system-reservoir coupling

$$W(t,t') \;=\; \left\{ \begin{array}{c} \text{sum of } \textit{irreducible} \text{ diagrams} \\ \text{for time-evolution} \end{array} \right.$$

[König et al., 1996, Schoeller, 2009, Leijnse and Wegewijs, 2008, Koller et al., 2010]

• Real-time diagrams + Liouville-space + quantum fields

$$W(t,t') = \sum_{\pm}$$



- **2$^{\text{nd}}$ quantization for *mixed quantum states***
  [Saptsov and Wegewijs, 2012] → cf. T. Prosen

$$(G^{+})^{\dagger} \;=\; G^{-} \qquad\qquad ((-\mathbb{1})^{N}|G^{-} \;=\; 0 \qquad\qquad G^{-}|\,\mathbb{1}\,) \;=\; 0$$

causal structure   superselection operator $T=\infty$ mixed state = vacuum

- ***Non-equilibrium* renormalization group for *open* systems** [Schoeller, 2009]
  Kondo: [Pletyukhov and Schoeller, 2012, Klochan et al., 2013]
  Decay: [Pletyukhov et al., 2010, Kennes et al., 2013]
  Pumping [Kashuba et al., 2012]

Surprising new insights open quantum systems: e.g. duality decay modes ↔ amplitudes

$$\boxed{\left[W(\omega; L, L^{\text{tun}})\right]^{\dagger} \;=\; \mathcal{P}\left[\;-\Gamma \;+\; W(i\Gamma-\omega^{*}; -L, i\,L^{\text{tun}})\;\right]\mathcal{P}, \qquad \mathcal{P}\bullet=(-1)^{N}\bullet}$$

Non-equilibrium + strong wide-band coupling + any T ! [Saptsov, Schulenborg]

• Kinetic equation - instantaneous solution

Simplicity: ρ diagonal in dot energy representation (no "coherences"): $L \to 0$

$$\partial_t|\rho(t)) \;=\; \int_{\underset{\uparrow}{-\infty}}^{t} \mathrm{d}t'\, W(t,t')\,|\rho(t'))$$

$\rho(t)$: stationary, time-dependent

Frozen parameters at value $R = R(t)$

$$\partial_t|\rho(t)) \;=\; 0 \;\overset{!}{=}\; W[\underset{\substack{\uparrow\\ \text{fixed}}}{R}]\cdot|\rho^i(t)) \quad \text{stationary state}$$

$$W[R] \;=\; \int_{-\infty}^{t} \mathrm{d}t'\, W[R](t-t') \;=\; \lim_{\omega\to i0} W[R](\omega) \quad \text{zero-frequency kernel}$$

Instantaneous reference solution

$$\boxed{\rho^i[R(t)] \;\; \text{parametric time dependence} \qquad \partial_t|\rho^i[R(t)]) \;\sim\; \Omega}$$

• Kinetic equation - adiabatic expansion

$$\partial_t |\rho(t)) = \int_{-\infty}^{t} \mathrm{d}t' \, W(t,t') \, |\rho(t'))$$

Expand "back into the past"

- state history around "memory time" $t' = t$

$$\rho(t') = \rho(t) + (t'-t)\partial_t \rho(t) + \ldots$$

- parameter history around "memory time" $\tau = t$

$$R(\tau) = R(t) + (\tau - t)\dot{R}(t) + \ldots$$

kernel functional:

$$W(t,t') = \underbrace{W(t-t')[R(t)]}_{\substack{\text{instantaneous kernel} \\ \text{frozen parameter}}} + \underbrace{W^a(t-t')\big[R(t), \dot{R}(t)\big]}_{\substack{\text{adiabatic-response} \\ \text{kernel}}} + \ldots$$

[Splettstoesser et al., 2006, Reckermann, 2010]

• Kinetic equation - adiabatic, weak coupling, high-temperature

$$\partial_t |\rho(t)) \;=\; \int_{-\infty}^{t} \mathrm{d}t' \, W(t,t') \, |\rho(t'))$$

$$|\rho(t)) \;=\; |\rho^i[R(t)]) + |\rho^a(t)), \qquad \partial_t |\rho^i[R(t)]) \;\sim\; \Omega$$

Adiabatic, weak coupling, high temperature: $\Omega \ll \Gamma \ll T$

$$\boxed{\begin{array}{c} \partial_t \, |\rho(t)) \;\approx\; W[R(t)] \, \rho(t) \\ W[R] \;:=\; \displaystyle\int_{-\infty}^{t} \mathrm{d}t' W(t-t')[R] \;=\; \lim_{\omega \to i0} W(\omega)[R] \end{array}}$$

Adiabatic + tunnel coupling expansion:

$$\partial_t \Big\{ \underset{\substack{\uparrow \\ \text{keep}}}{\overset{\substack{\Omega \\ \downarrow}}{|\rho^i)}} + \overset{\substack{\Omega \times ? \\ \downarrow}}{|\rho^a(t))} \Big\} \;=\; \int_{-\infty}^{t} \mathrm{d}t' \Big[\underset{\substack{\uparrow \\ \text{keep}}}{\overset{\substack{\Gamma \\ \downarrow}}{W(t-t')}} + \overset{\substack{\Gamma \cdot (\Omega / T) \\ \downarrow}}{W^a(t-t')} \Big] \cdot \Big[\overset{\substack{1 \\ \downarrow}}{1} + \overset{\substack{\Omega / T \\ \downarrow}}{(t'-t)\frac{\partial}{\partial t}} \Big] \cdot \Big[\underset{\substack{\uparrow \\ \text{keep}}}{\overset{\substack{1 \\ \downarrow}}{|\rho^i(t))}} + \underset{\substack{\uparrow \\ \text{keep}}}{\overset{?}{|\rho^a(t))}} \Big]$$

• Nonlocal observables

Any observable

$$A(t) \;=\; \langle A\rangle^{\mathrm{res}}(t) + \mathbf{:}A(t)\mathbf{:}$$

Stationary, time-dependent expectation:

$$\begin{aligned}
\boxed{\langle A\rangle(t)} &= \mathrm{Tr}\,\mathrm{Tr}_{\mathrm{res}}\, A(t)\, e^{-i\int_{-\infty}^{t} d\tau L^{\mathrm{tot}}(\tau)}\, \rho^{\mathrm{res}}\, \rho(-\infty) \\
&= \mathrm{Tr}\,\langle A(t)\rangle^{\mathrm{res}} \rho(t) + \mathrm{Tr}\int_{-\infty}^{t} \mathrm{d}t'\, W_{\mathbf{:}A\mathbf{:}}(t,t')\, \rho(t') \\
&= \boxed{\mathrm{Tr}\int_{-\infty}^{t} \mathrm{d}t'\, W_{A}(t,t')\, \rho(t')}
\end{aligned}$$

↑
diagrams
similar to W

Observable kernel:

$$W_A(t,t') \;:=\; \langle A(t)\rangle^{\mathrm{res}}\, \delta(t-t'-0) + W_{\mathbf{:}A\mathbf{:}}(t,t')$$

• Nonlocal observables - adiabatic expansion

Adiabatic, weak coupling, high temperature: $\Omega \ll \Gamma \ll T$

$$\boxed{\begin{aligned} \langle A\rangle(t) \;&\approx\; (\mathbb{1}|\,W_A[R(t)]\,|\rho(t)) \\ W_A[R] \;:=\; \int_{-\infty}^{t} \mathrm{d}t' W_A(t-t')[R] \;&=\; \langle A(t)\rangle^{\mathrm{res}}[R] + \lim_{\omega\to i0} W_{:A:}(\omega)[R] \end{aligned}}$$

Shifting property of zero-frequency kernel:

$$\boxed{W_{A+g\,\mathbb{1}^{\mathrm{tot}}}[R] \;=\; g\mathcal{I} + W_A[R]}$$

- $\langle g\mathbb{1}^{\mathrm{tot}}\rangle^{\mathrm{res}} = g\mathcal{I}$
- $:A + g\,\mathbb{1}^{\mathrm{tot}}: \;=\; :A:$

• Geometry of pumping is NOT in the quantum state ...

$$\boxed{\partial_t \rho(t) \; = \; W[R(t)] \,|\, \rho(t)), \qquad \left\langle \frac{\mathrm{d}\widehat{N}^r}{\mathrm{d}t} \right\rangle(t) \; = \; (\mathbb{1}|\, W_{\frac{\mathrm{d}\widehat{N}^r}{\mathrm{d}t}}[R(t)] \,|\rho(t))}$$

$$\rho(t) \; \approx \; \rho^i[R(t)] + \rho^a\big[R(t), \dot{R}(t)\big]$$

Adiabatic-response solution: linear in driving rates $\dot{R}(t)$

$$0 \; = \; W[R(t)]\, \rho^i \quad \longrightarrow \quad \partial_t\, \rho^i[R(t)] \; \approx \; W[R(t)]\, \rho^a$$

Stationary quantum state → $\boxed{\rho(t) \text{ has } \textbf{no} \text{ geometric phase}}$
but **nonstationary** components of quantum state **do** have geometric “Berry” phases !
→ [Sarandy and Lidar, 2005, Sarandy and Lidar, 2006] + adiabatic iteration

Current:

$$\left\langle \frac{\mathrm{d}\widehat{N}^r}{\mathrm{d}t} \right\rangle(t) \; \approx \; \underbrace{(\mathbb{1}|W_{\frac{\mathrm{d}\widehat{N}^r}{\mathrm{d}t}}[R(t)]\,|\rho^i[R(t)])}_{\text{instantaneous}} + \underbrace{(\mathbb{1}|W_{\frac{\mathrm{d}\widehat{N}^r}{\mathrm{d}t}}[R(t)]\, \frac{1}{W[R(t)]}|\partial_R\, \rho^i[R(t)]) \cdot \dot{R}}_{\text{pumping}}$$

• Geometry of pumping is NOT in the quantum state ...

$$\partial_t \rho(t) = W[R(t)]\, \rho(t)$$

- Frozen parameter solution $\partial_t \rho(t) \overset{!}{=} 0 = W \rho(t) = 0$
  $\rightarrow$ long time solution with **zero** geometric phase since $(\mathbb{1}|\rho(\tau)) = 1$ all τ

$$\rho(t) \approx \rho^i[R(t)]$$

  Full adiabatic dynamics: $v_0 := \rho^i$ zero eigenvector with $(\bar{v}_0|\partial_t v_0(\tau)) = 0$

$$W(t) = \sum_i \lambda_i(t)\, |v_i(t))(\bar{v}_i(0)| \;\Rightarrow\; \Pi(t) = \sum_i e^{\int_0^t \mathrm{d}\tau\, [\lambda_i(\tau) - (\bar{v}_i(\tau)|\partial_t v_i(\tau))]}\, |v_i(t))(\bar{v}_i(0)|$$

- Frozen parameter R + frozen velocity $\dot{R}$ solution in adiabatic frame

$$\partial_t|\rho'(t)) \overset{!}{=} 0 = \Pi(t)^{-1}\Big(W(t) - (\partial_t \Pi(t))\, \Pi(t)^{-1}\Big)\Pi(t)\, |\rho'(t)) = 0$$

  $\rightarrow$ long time solution with **zero** geometric phase since $(\mathbb{1}|\rho'(\tau)) = 1$ all τ

$$|\rho(t)) = \Pi(t)|\rho'(t)) \approx |\rho^i[R(t)]) + \frac{1}{W(t) - (\partial_t \Pi(t))\, \Pi(t)^{-1}} \partial_t\, |\rho^i[R(t)])$$

$$\overset{\Omega \ll \Gamma}{\approx} |\rho^i[R(t)]) + \frac{1}{W(t)} \partial_t\, |\rho^i[R(t)])$$

• Pumped charge

$$\int_0^T \mathrm{d}t \left\langle \frac{\widehat{\mathrm{d}N}^r}{\mathrm{d}t} \right\rangle(t) \;=\; Q^r \;=\; Q^{r,i} + Q^{r,a}$$

- **Instantaneous transferred charge** = "dynamical part"

$$Q^{r,i} \;=\; \int_0^T \mathrm{d}t\, (\mathbb{1}|\, W_{\frac{\widehat{\mathrm{d}N}^r}{\mathrm{d}t}}[R(t)]\,|\rho^i[R(t)])$$

- **Pumped charge** = "geometric part"

$$\boxed{\begin{array}{lllll} Q^{r,a} = \oint_C \mathrm{d}R\, A^r[R] & \leftarrow & A^r[R] = (\,\varphi^r[R]\,|\,\partial_R\, \rho^i[R]\,) & \text{gauge potential} \\ \quad\;\; = \int_{\mathcal{S}} \mathrm{d}\mathcal{S}\, B^r[R] & \leftarrow & B^r[R] = (\partial_R\, \varphi^r[R]\,|\times|\,\partial_R\, \rho^i[R]\,) & \text{gauge field} \end{array}}$$

Adiabatic-response covector

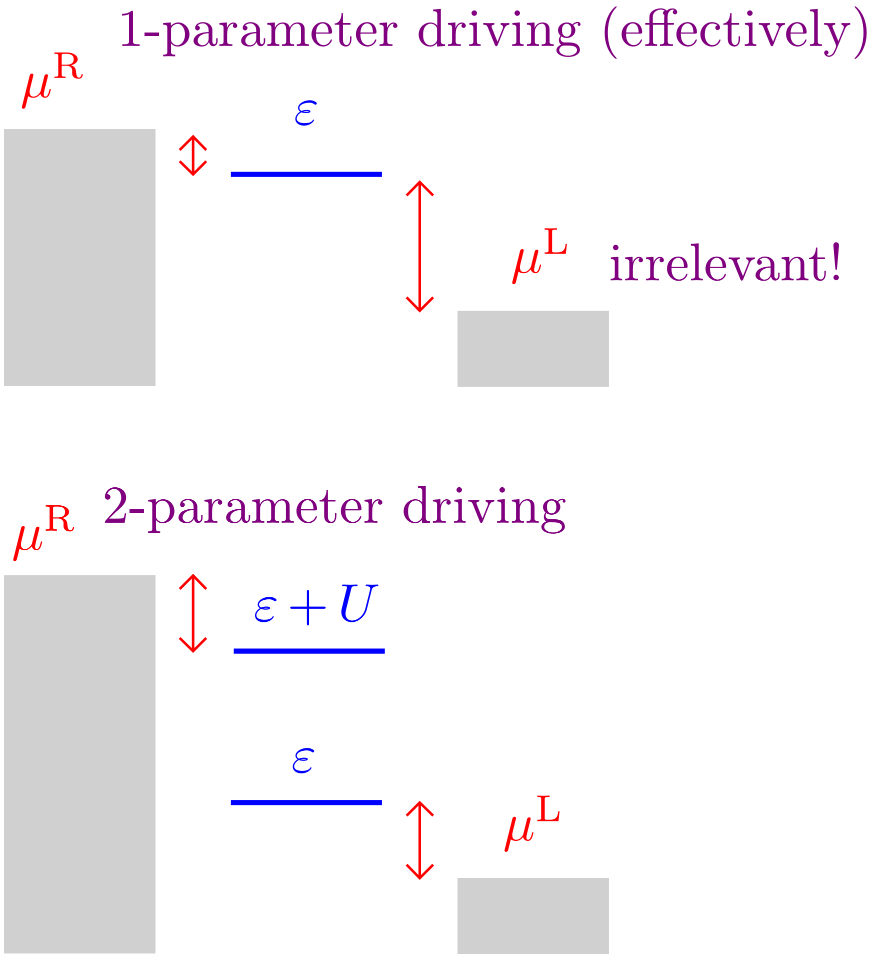
[Calvo et al., 2012, Avron et al., 2012, Büttiker et al., 1993, Brouwer, 1998]

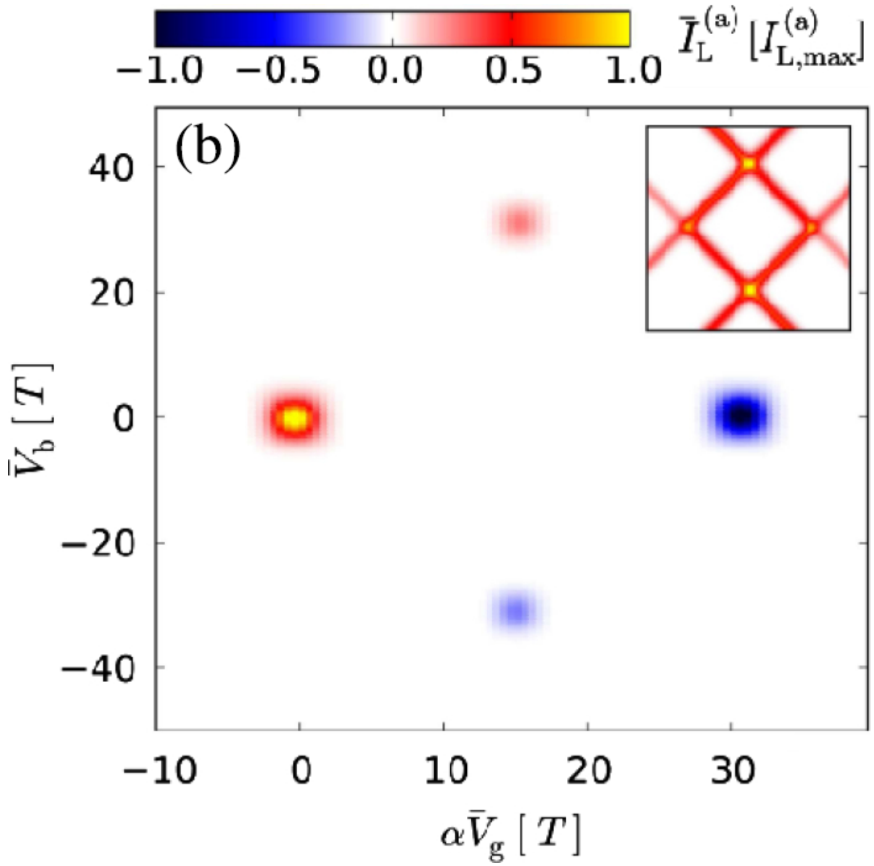
$$\boxed{(\,\varphi^r[R]\,|\bullet \;=\; (\mathbb{1}|\underbrace{W_{\frac{\widehat{\mathrm{d}N}^r}{\mathrm{d}t}}[R]}_{\text{current}}\underbrace{\frac{1}{W[R]}}_{\text{relaxation time}}\bullet}$$

• Conditions for pumping - 2-parameter driving

$$\boxed{B^r(R) \;=\; (\partial_R\,\varphi^r[R]|\times|\partial_R\,\rho^i[R]) \;=\; \frac{\text{pumped charge}}{\text{parameter area}} \qquad r=\mathrm{L,R}}$$

Pumping @ *crossing of frozen-parameter resonances* different electrodes





• Conditions for pumping - asymmetric driving

$$\boxed{B^r(R) \;=\; (\partial_R\, \varphi^r[R]| \times |\partial_R\, \rho^i[R]) \;=\; \frac{\text{pumped charge}}{\text{parameter area}} \qquad r = \mathrm{L,R}}$$

Spatial asymmetric driving is required for pumping conserved observable

1. Stationary current conservation for *frozen-parameters*

$$\sum_r \left\langle \frac{\widehat{\mathrm{d}N^r}}{\mathrm{d}t} \right\rangle^a (t) \;=\; \int_S \mathrm{d}S\,(B^{\mathrm{L},a} + B^{\mathrm{R},a}) \;=\; 0 \;\; \text{any driving curve } \partial S$$

2. Junction-antisymmetrized pumped current:

$$(B^{r,\mathrm{L}} - B^{r,\mathrm{R}}) \;=\; \underbrace{(\partial_R\, \tfrac{1}{2}(\varphi^L - \varphi^R)|}_{(N|\partial_R \frac{1}{2}(W^{\mathrm{L}} - W^{\mathrm{R}})\,\frac{1}{W} \atop \text{antisymmetric}} \times \underbrace{|\partial_R\, \rho^i[R])}_{(W^{\mathrm{L}} + W^{\mathrm{R}})\rho^i = 0 \atop \text{symmetric equation}} \qquad \leftarrow \;\; \text{non-parallel vectors}$$

• Conditions for pumping - interaction !

$$\boxed{B^r(R) \;=\; (\partial_R \varphi^r[R]| \times |\partial_R \rho^i[R]) \;=\; \frac{\text{pumped charge}}{\text{parameter area}} \qquad r = \mathrm{L,R}}$$

Interaction is required for pumping

- Conserved observable

$$(\varphi^r[R]|\bullet \;=\; \underset{\text{current junction } r}{\underset{\uparrow}{(\mathbb{1}|W_{\widehat{\frac{dN^r}{dt}}}[R]}} \underset{\text{relaxation time}}{\underset{\uparrow}{\frac{1}{W[R]}}}\bullet \quad \overset{\downarrow}{=} \quad \underset{\text{charge}}{\underset{\uparrow}{-(N|}} \underset{\text{fraction through junction } r}{\underset{\uparrow}{W^r[R]}} \frac{1}{W[R]}\bullet$$

- Split up $T = \infty$ and $T < \infty$ contributions

$$W \;=\; \sum_r W^r, \qquad W^r \;=\; W^r|_{T=\infty} + \underset{\text{drops out !}}{\boxed{\Delta W^r|_{U=0}}} + \Delta W^r\big|_{U\neq 0}$$

$$\boxed{-(\partial_R \varphi^r| \;=\; (N|\partial_R W^r \frac{1}{W}|_{T=\infty} + (N|\partial_R\left\{1 - W^r \frac{1}{W}|_{T=\infty}\right\}\Delta W\big|_{U\neq 0} \cdot \frac{1}{W}}$$

↑

depends on $\Sigma_r \Gamma_{r\sigma l}$ only for each spin σ + orbital l

• Gauge freedom - current-equivalent observables

- **Global gauge freedom**: same, constant g for all parameters R

$$\hat{N}^r \;\to\; \hat{N}^r_g \;:=\; \hat{N}^r + g\,\mathbb{1}^{\mathrm{tot}}, \qquad \frac{\mathrm{d}\widehat{N}^r}{\mathrm{d}t} \;=\; i\big[H^{\mathrm{tot}}, \hat{N}^r\big] + \frac{\partial \hat{N}^r}{\partial t}$$

Observables $\hat{N}^r \sim \hat{N}^r_g$ equivalent $\Longleftrightarrow$ **equal** current (super) operators

$$W_{\widehat{\frac{\mathrm{d}N^r_g}{\mathrm{d}t}}}[R] \;=\; W_{\widehat{\frac{\mathrm{d}N^r}{\mathrm{d}t}}}[R]$$

- **Local gauge freedom**: continuous parameter function $g(R)$

$$\hat{N}^r \;\to\; \hat{N}^r_{g(R)} \;:=\; \hat{N}^r + g(R)\,\mathbb{1}^{\mathrm{tot}} \qquad \frac{\mathrm{d}\widehat{N_g}^r}{\mathrm{d}t} \;=\; \frac{\mathrm{d}\widehat{N}^r}{\mathrm{d}t} + \partial_t g[R(t)]\,\mathbb{1}^{\mathrm{tot}}$$

Observables $\hat{N}^r \sim \hat{N}^r_g$ equivalent $\Longrightarrow$ **different** “gauged” current (super) operators

$$W_{\widehat{\frac{\mathrm{d}N^r_g}{\mathrm{d}t}}}[R(t)] \;=\; W_{\widehat{\frac{\mathrm{d}N^r}{\mathrm{d}t}}}[R(t)] + \partial_t\, g[R(t)]\,\mathcal{I}$$

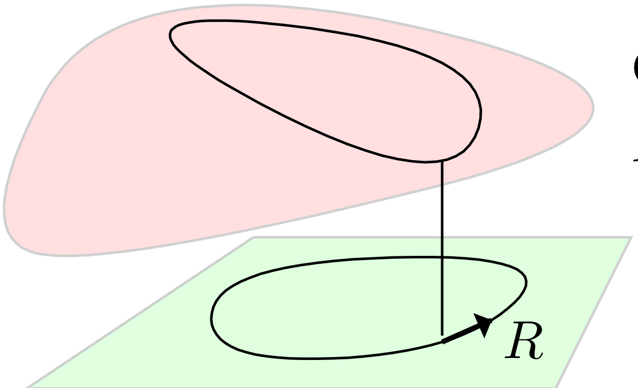
• Gauge freedom - current-equivalent observables

Observables $\hat{N}^r \sim \hat{N}_g^r$ equivalent $\Longrightarrow$ **equal** *pumped charges* !

$$Q = \int_0^T \mathrm{d}t \left\langle \widehat{\frac{\mathrm{d}N^r}{\mathrm{d}t}} \right\rangle(t) \overset{!}{=} \int_0^T \mathrm{d}t \left\langle \widehat{\frac{\mathrm{d}N_g{}^r}{\mathrm{d}t}} \right\rangle(t) + \oint \mathrm{d}R \; \partial_R \, g(R)$$

$\boxed{\text{Pumped quantities} \to \text{local gauge freedom}}$

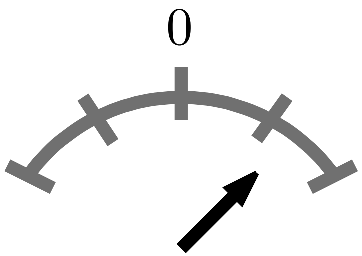
Geometry: **fiber-bundle** of observables (g) $\times$ parameters (R)



Observables

$N^r_{g(R)} = N^r + g(R)\mathbb{1}$

Parameter manifold



Gauge g = continuous choice of scale-bar on pumped-N^r meter

• Connection = pumping current

$$(\mathbb{1}|\rho^i[R(t)]) \;=\; 1 \qquad\qquad (\mathbb{1}|\rho^a\big[R(t),\dot{R}(t)\big]) \;=\; 0$$

Current for gauged observable: **pumping** part ($\propto \dot{R}$) is **gauge dependent** ($\propto \dot{g} = \partial_R g\, \dot{R}$)

$$\left\langle \widehat{\frac{\mathrm{d}N_g^r}{\mathrm{d}t}} \right\rangle \approx (\mathbb{1}|\left\{W_{\widehat{\frac{\mathrm{d}N^r}{\mathrm{d}t}}} + \partial_t\, g[R(t)]\,\mathcal{I}\right\} \cdot \left\{|\rho^i[R]) + |\rho^a\big[R,\dot{R}\big])\right\}$$

$$= \underbrace{(\mathbb{1}|W_{\widehat{\frac{\mathrm{d}N^r}{\mathrm{d}t}}}[R]\,|\rho^i[R])}_{\text{instantaneous = invariant}} \;+\; \underbrace{\dot{g} + (\mathbb{1}|W_{\widehat{\frac{\mathrm{d}N^r}{\mathrm{d}t}}}[R]\,|\rho^a\big[R,\dot{R}\big])}_{\text{pumping = gauge dependent}} \leftarrow \left\langle \widehat{\frac{\mathrm{d}N_g^r}{\mathrm{d}t}} \right\rangle^a$$

Bilinear function in tangent vectors $\dot{R}$ and $\dot{g}$ in total space of fiber bundle:
⇒ **geometric connection**: defines "horizontal lift" / parallel transport

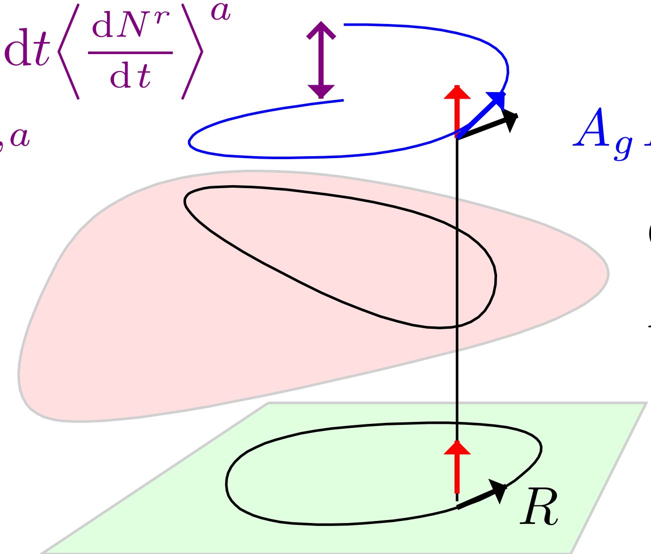
$$\left\langle \widehat{\frac{\mathrm{d}N_g^r}{\mathrm{d}t}} \right\rangle^a (\dot{R},\dot{g}) \;=\; 0 \quad\Longleftrightarrow\quad (R,g) \text{ curve "horizontal"}$$

• Connection = Landsberg type (1992)

Holonomy:

$$g(T) - g(0) = -\int \mathrm{d}t \left\langle \frac{\mathrm{d}N^r}{\mathrm{d}t} \right\rangle^a = -Q^{r,a}$$

Horizontal lift

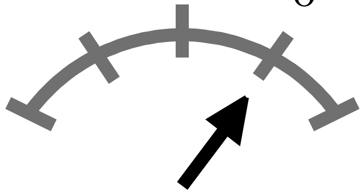


$N^{r,a} \stackrel{!}{=} 0$ (connection rule)

$A_g \dot{R} = \dot{g} + \left\langle \frac{\mathrm{d}N^r}{\mathrm{d}t} \right\rangle^a \stackrel{!}{=} 0$ (connection 1-form)

Observables

$N^r_{g(R)} = N^r + g(R)\mathbb{1}$



Parameter manifold

Landsberg geometric phase $\neq$ "Berry phase" !

- dissipative system with **stationary state** + a symmetry (equivalence)
- adiabatic-response (**nonadiabatic** correction)

[Landsberg, 1992, Landsberg, 1993, Andersson, 2003a, Andersson, 2003b]

$$\partial_t \,|\rho(t)) \;=\; W[R(t)]\,|\rho(t)) \quad \text{master} \qquad \left\langle \frac{\widehat{\mathrm{d}N^r}}{\mathrm{d}t} \right\rangle(t) \;=\; (\mathbb{1}|W_{\frac{\widehat{\mathrm{d}N^r}}{\mathrm{d}t}}[R(t)]\,|\rho(t)) \quad \text{slave}$$

• Interaction-induced geometric pumping

1. **Nonadiabatic, nonequilibrium pumping formula**:

$$B^r(R) = (\partial_R \varphi^r[R]| \times |\partial_R \rho^i[R]) = \frac{\text{pumped charge}}{\text{parameter area}} \qquad r = \text{L,R}$$

$$(\varphi^r[R]|\bullet = (\mathbb{1}|W_{\widehat{\frac{dN}{dt}}^r}[R]\frac{1}{W[R]}\bullet \qquad \text{response covector}$$

Interaction required *when "bare" coupling not driven* $(\Sigma_r \Gamma^r_{l\sigma})$

2. **Geometry**:

$$\text{Fiber-bundle} = \text{observables} \times \text{parameters}$$

Gauge freedom = observable - current non-uniqueness
cf. [Sinitsyn, 2009, Avron et al., 2012]

$$\hat{N}^r \xrightarrow{\text{many-to-1}} \widehat{\frac{dN^r}{dt}} = i[H, \hat{N}^r] + \frac{\partial}{\partial t}\hat{N}^r$$

Connection = pumping current
Holonomy = pumped charge ← Landsberg "phase" ≠ *Berry-Simon* "phase"

(1) Phys. Rev. Lett. *104*, 226803 (2010) (2) Phys. Rev. B *86*, 245308 (2012)
(3) T. Plücker, H. Calvo, M. R. Wegewijs, J. Splettstoesser, *in preparation*

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