On ‘Solving’ a quantum many body problem by experiment

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T. Schweigler et al. arXiv:1505.03126
A. Steffens et al. nat. comm. (2015) arXiv:1406.3632

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Quantum fields <-> Correlation functions

On the Green's functions of quantized fields
J. Schwinger PNAS (1951)

- Solving a quantum many-body problem is equivalent to knowing **all** its correlation functions.
- In practice, an observer can only measure a **finite** number of correlations describing the propagation and scattering of excitations.
- To solve a problem one need to **find degrees of freedom** where only few (low order) correlation functions are relevant.
- If one finds the degrees of freedom (basis) where the **correlation functions factorize**, this is equivalent to **diagonalization of the many body Hamiltonian.**

Outline

1d System

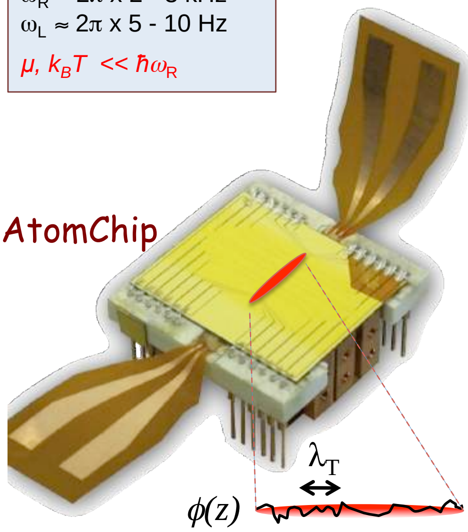
Correlation functions
- fields <-> phase <-> excitations

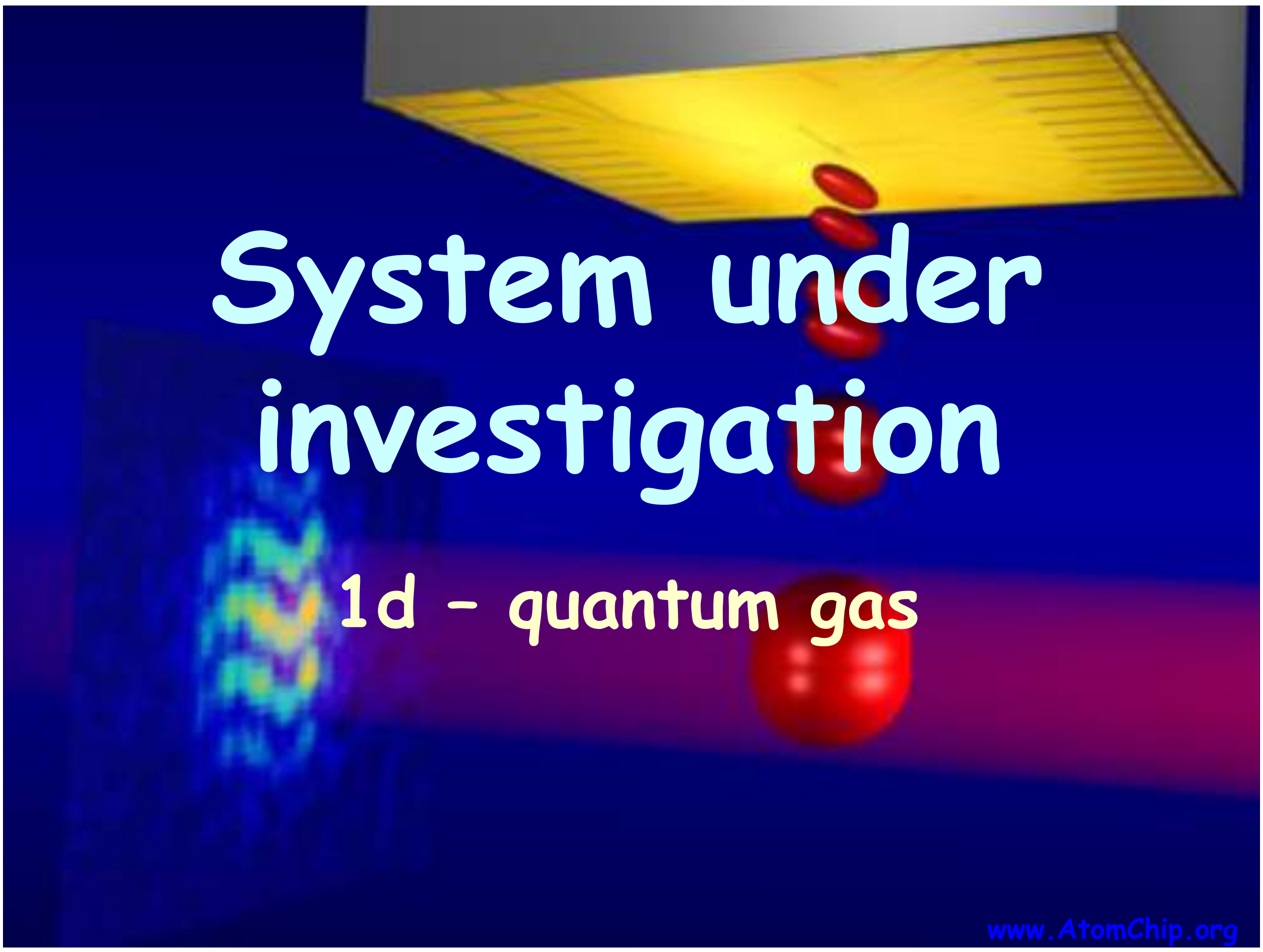
High order correlation functions
- Quantifying factorization
- Sine-Gordon model
- Quench to a free system

Quantum Field Tomography

Outlook
- entanglement and spin squeezing
- relaxation in SG model

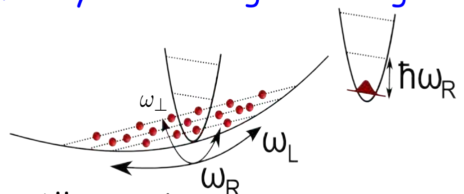
1000-10000 Rb atoms
T = 10-100 nK
$\omega_R \approx 2\pi \times 2 - 3$ kHz
$\omega_L \approx 2\pi \times 5 - 10$ Hz
$\mu, k_B T << \hbar\omega_R$





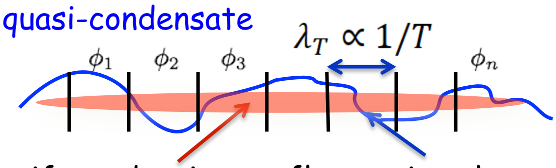
System under investigation

Weakly interacting 1d Bose gas



All energies $\mu, k_B T << \hbar\omega_\perp$

quasi-condensate



uniform density fluctuating phase

$$\hat{\psi}(x) = e^{i\hat{\Phi}_1(x)}\sqrt{\rho + \hat{n}_1(x)}$$

thermally populated

Lieb-Liniger model

- Exactly solvable integrable theory

low energy effective field theory:

Luttinger-liquid

$$H = \frac{c}{2}\int dx \left[\frac{K}{\pi}(\nabla\hat{\varphi})^2 + \frac{\pi}{K}\hat{n}^2\right]$$

- excitations are soundwaves (phonons)
- linear dispersion relation

coupled 1d systems:

Sine-Gordon model

$$\hat{H}_{SG} = \frac{\hbar c}{2}\int_{-L/2}^{L/2} dz \left[\frac{\pi}{K}\hat{n}^2(z) + \frac{K}{\pi}\left(\frac{\partial}{\partial z}\hat{\theta}(z)\right)^2\right] - 2n_{1D}J\int_{-L/2}^{L/2} dz \cos[\sqrt{2}\,\hat{\theta}(z)]$$

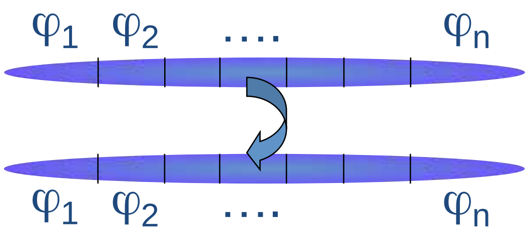
Model for interacting many body systems which can be described by a field theory with long lived excitations.

The longitudinal phase fluctuations are key for our experiments

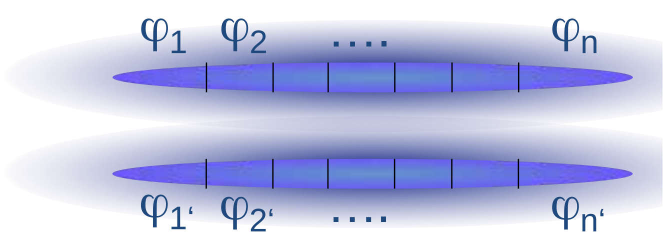
interference of phase fluctuating 1D condensates

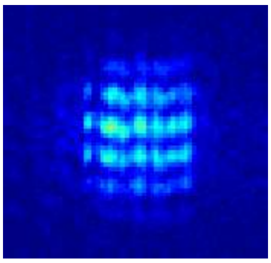
Study the quantum field, its excitations and relaxation

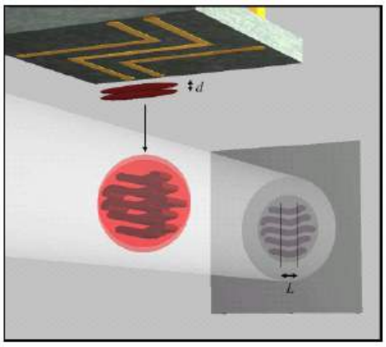
create a copy by splitting
quantum connected

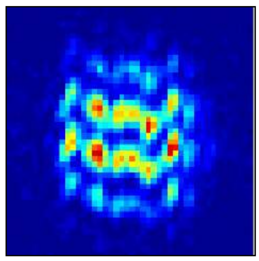


create two independet samples
classically separated



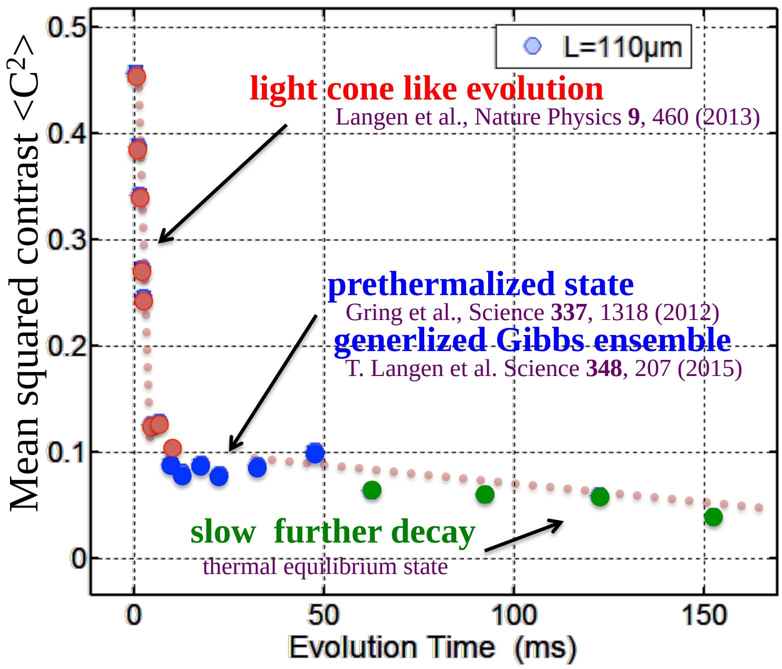


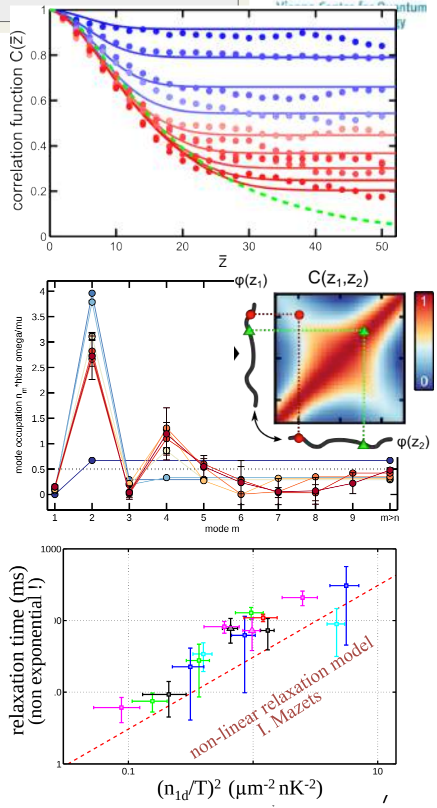




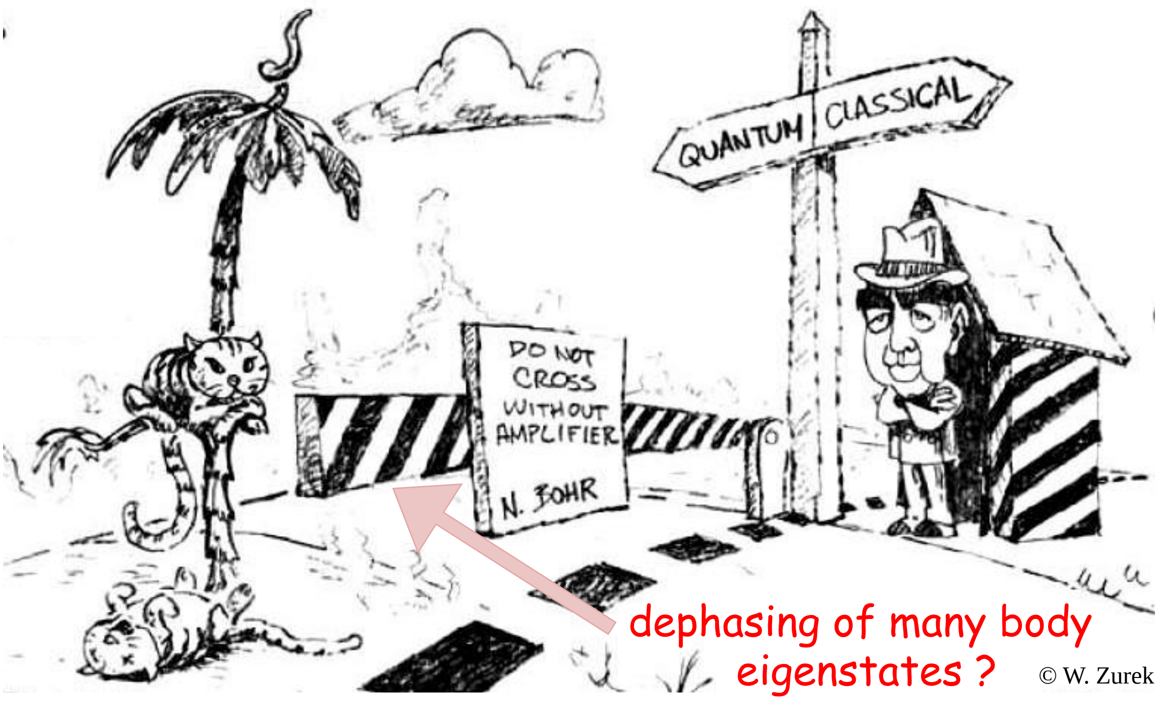
Evolution after the quench

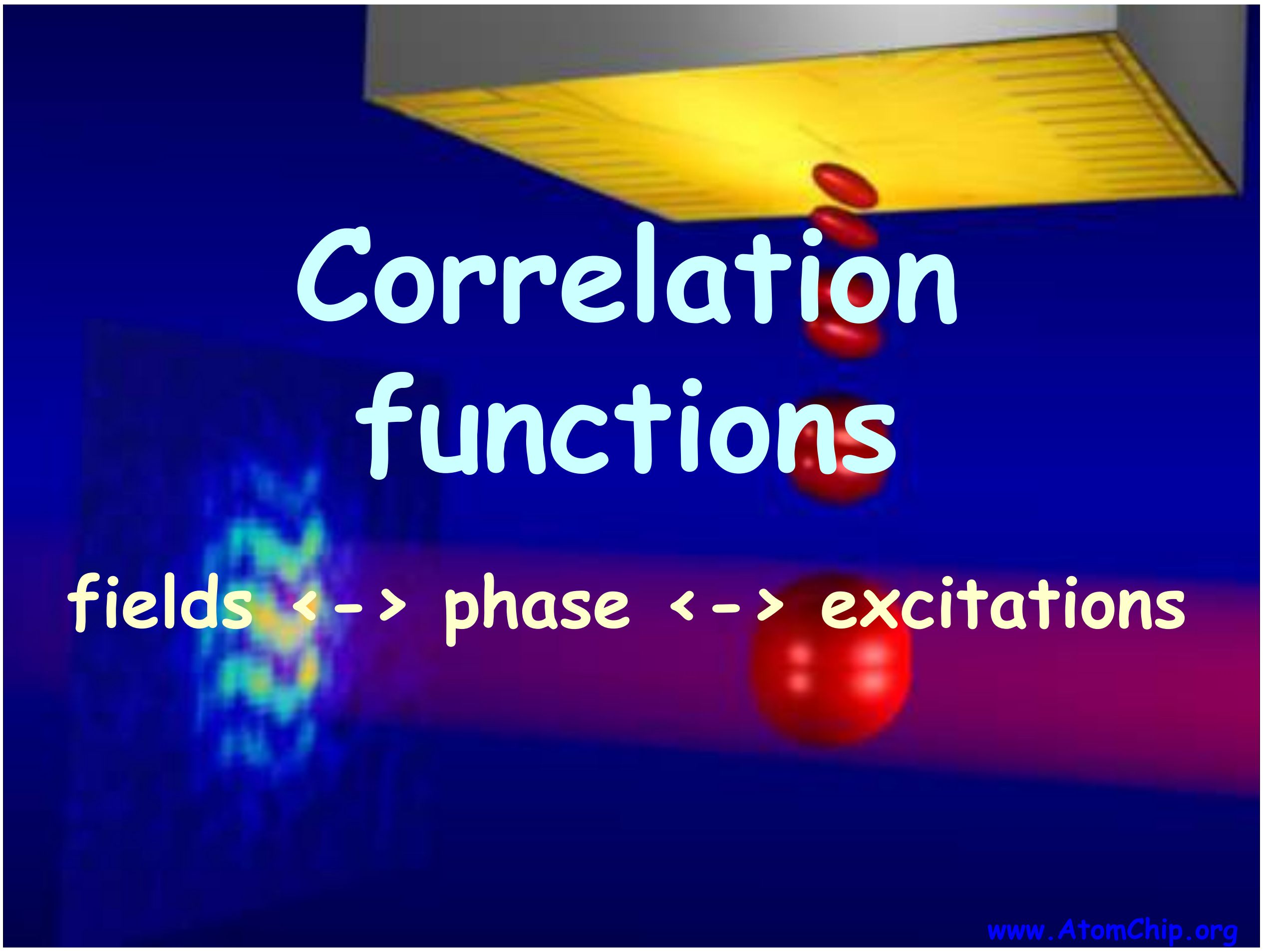
Decay of the mean contrast





Emergence of classical world from quantum evolution





# Correlation functions
fields <-> phase

T. Schweigler et al. arXiv:1505.03126

experiments in a trap
-> non translation invariant correlation functions

$$C(z_1, z_2) = \frac{\langle \Psi_1(z_1)\Psi_2^\dagger(z_1)\Psi_1^\dagger(z_2)\Psi_2(z_2)\rangle}{\langle|\Psi_1(z_1)|^2\rangle\langle|\Psi_2(z_2)|^2\rangle}$$

with
$$\Psi(z) = e^{i\theta(z)}\sqrt{\rho_0(z) + \delta\hat{n}(z)}$$
$$\varphi(z) = \theta_1(z) - \theta_2(z)$$

neglecting $\delta\hat{n}(z)$ ⇒ $\boxed{C(z_1, z_2) \approx \langle \exp[i\varphi(z_1) - i\varphi(z_2)]\rangle}$

4^{th} order:

$$C(z_1, z_2, z_3, z_4) = \frac{\langle \Psi_1(z_1)\Psi_2^\dagger(z_1)\Psi_1^\dagger(z_2)\Psi_2(z_2)\Psi_1(z_3)\Psi_2^\dagger(z_3)\Psi_1^\dagger(z_4)\Psi_2(z_4)\rangle}{\langle|\Psi_1(z_1)|^2\rangle\langle|\Psi_1(z_2)|^2\rangle\langle|\Psi_2(z_3)|^2\rangle\langle|\Psi_2(z_4)|^2\rangle}$$

$$C(z_1, z_2, z_3, z_4) \approx \langle \exp[i\varphi(z_1) - i\varphi(z_2) + i\varphi(z_3) - i\varphi(z_4)]\rangle$$

Correlation functions
excitations <-> phase

T. Schweigler et al. arXiv:1505.03126

in experiment we measure the phase $\varphi(z)$ directly
-> look at phase correlators

$$C^{(2)}(z_1, z_2) = \langle[\varphi(z_1) - \varphi(z_2)]^2\rangle = \langle[\Delta\varphi(z_1,z_2)]^2\rangle$$

with $\Delta\varphi(z_1, z_2) = \varphi(z_1) - \varphi(z_2)$ Note: $\Delta\varphi$ is NOT restricted to 2π

using
$$\varphi(z) = \frac{1}{\sqrt{L}} \sum_{k\neq 0} \left[(-i)\sqrt{\frac{\pi}{|k|K}}(b_k^\dagger - b_{-k})e^{ikz}\right]$$

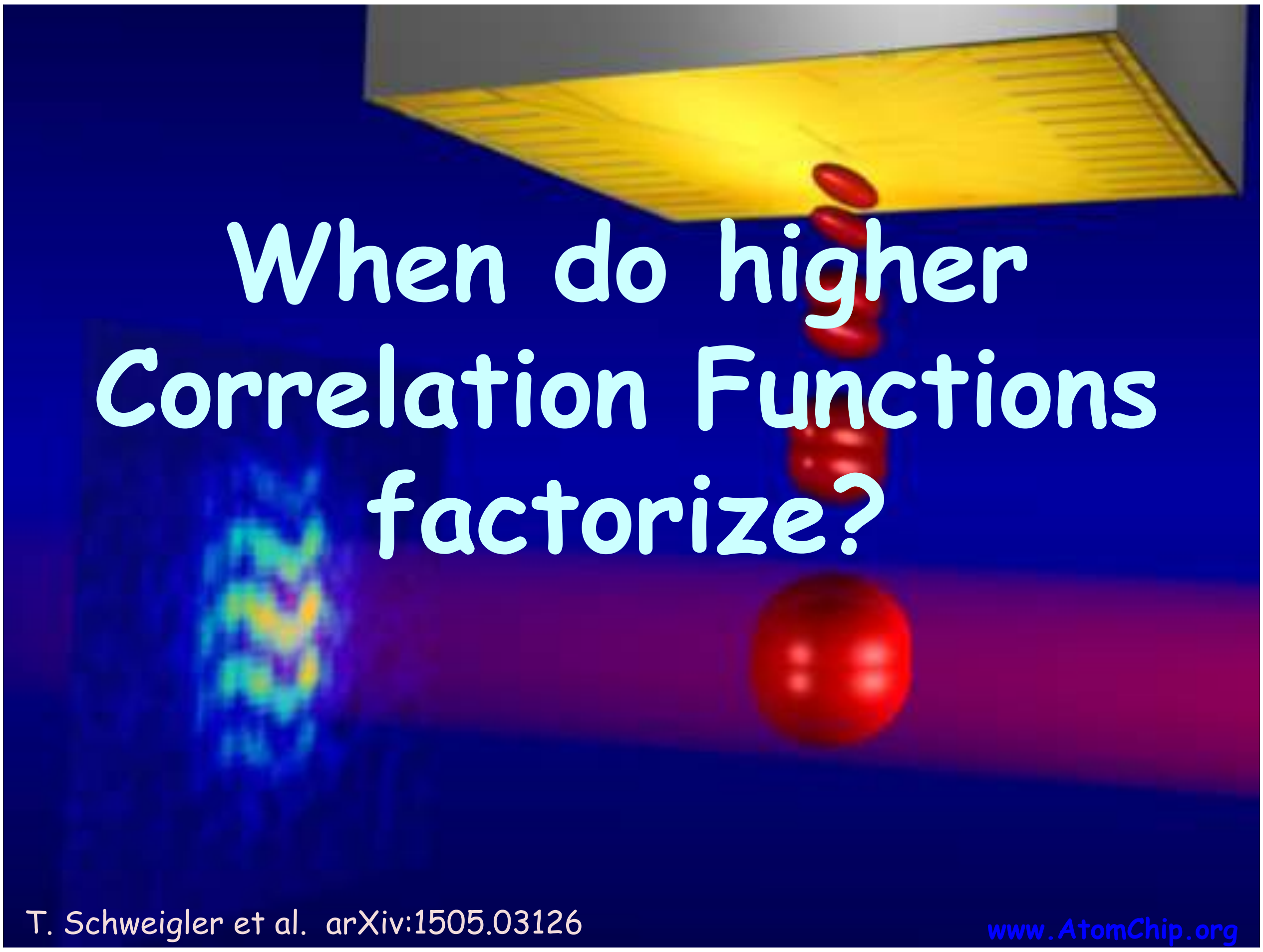
$$\Rightarrow \langle[\varphi(z_1) - \varphi(z_2)]^2\rangle = \sum_{k_1,k_2} \frac{\pi}{K\sqrt{|k_1 k_2|}} b_{k_1}^\dagger b_{-k_2} e^{ik_1 z_1 + ik_2 z_2} + \dots$$

-> phase correlators are related to the **quasi particles**

4th order

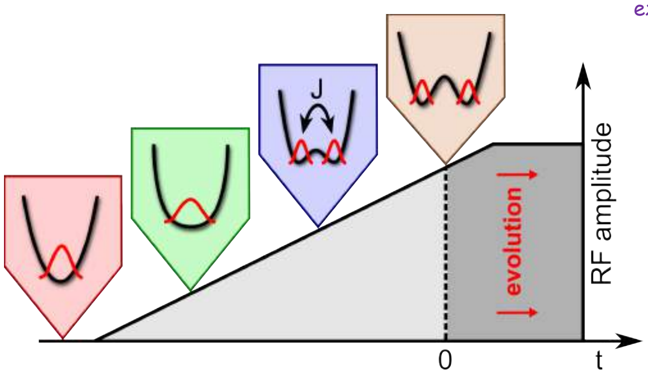
$$C^{(4)}(z_1, z_2, z_3, z_4) = \langle[\varphi(z_1) - \varphi(z_2)]^2 [\varphi(z_3) - \varphi(z_4)]^2\rangle$$
$$\propto b_{k_1}^\dagger b_{k_2}^\dagger b_{-k_3} b_{-k_4} + \dots$$

-> quasi particle scattering



Sine-Gordon physics
1d double-well with tunable coupling J

experiment: T. Schweigler et al.
theory: V. Kasper, S. Erne
T Gasenzer, J. Berges



Quantum Sine-Gordon model:

$$\hat{H}_{\rm SG} = \int dz \left[\frac{\hbar^2 n_{1\rm D}}{4m} (\partial_z \hat{\varphi})^2 + g \delta \hat{\rho}^2 \right] - \int dz \; 2 J n_{1D} \left[1 - \cos \hat{\varphi} \right]$$

phase coherence length

$$\lambda_T = 2\hbar^2 n_{1D} / (m k_B T)$$

phase (spin) healing length

$$l_J = \sqrt{\hbar / (4mJ)}$$

Characteristic parameters

$$q = \lambda_T / l_J$$

Sine-Gordon physics
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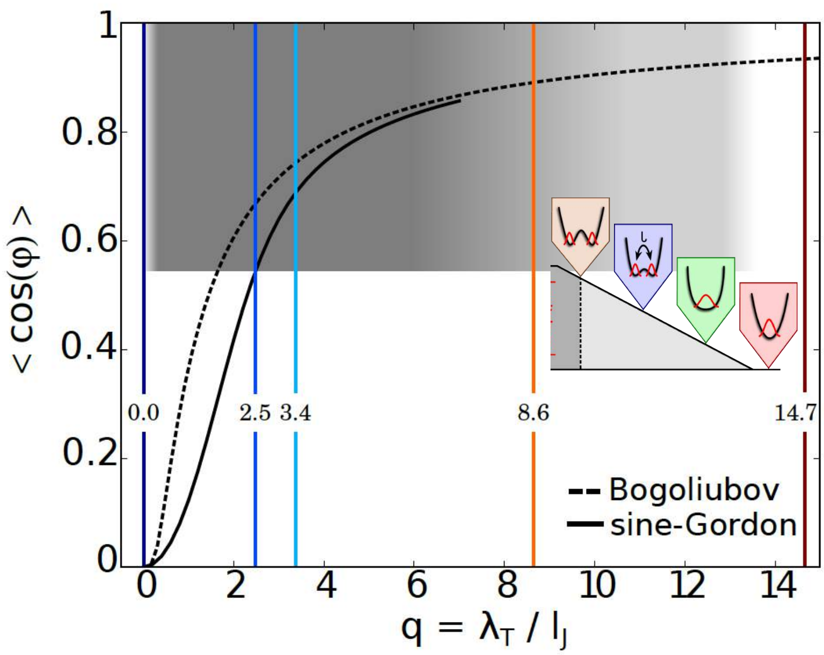


Characterising the factorisation

T. Schweigler et al. arXiv:1505.03126

experiments probe the phase

-> look at the **'connected part'** of the phase correlation function

Gaussian fluctuations

$$\langle(\Delta\varphi)^2\rangle_c = \langle(\Delta\varphi)^2\rangle \qquad \text{Variance}$$
$$\langle(\Delta\varphi)^4\rangle_c = \langle(\Delta\varphi)^4\rangle - 3\,\langle(\Delta\varphi)^2\rangle^2 \qquad = 0$$
$$\langle(\Delta\varphi)^6\rangle_c = \langle(\Delta\varphi)^6\rangle - 15\,\langle(\Delta\varphi)^4\rangle\,\langle(\Delta\varphi)^2\rangle^2 + 30\,\langle(\Delta\varphi)^2\rangle^3 \qquad = 0$$
$$\langle(\Delta\varphi)^8\rangle_c = \langle(\Delta\varphi)^8\rangle + 420\,\langle(\Delta\varphi)^4\rangle\,\langle(\Delta\varphi)^2\rangle - 630\,\langle(\Delta\varphi)^2\rangle^4 - 35\,\langle(\Delta\varphi)^4\rangle^2 - 28\,\langle(\Delta\varphi)^6\rangle\,\langle(\Delta\varphi)^2\rangle \qquad = 0$$

characterized by **'Kurtosis'**

Gaussian fluctuations

$$\gamma_2 = \frac{\langle(\Delta\varphi)^4\rangle}{3\,\langle(\Delta\varphi)^2\rangle^2} - 1 \qquad = 0$$

$$\gamma_3 = \frac{\langle(\Delta\varphi)^6\rangle}{15\,\langle(\Delta\varphi)^4\rangle\,\langle(\Delta\varphi)^2\rangle^2 - 30\,\langle(\Delta\varphi)^2\rangle^3} - 1 \qquad = 0$$

$$\gamma_4 = \frac{\langle(\Delta\varphi)^8\rangle}{630\,\langle(\Delta\varphi)^2\rangle^4 + 35\,\langle(\Delta\varphi)^4\rangle^2 + 28\,\langle(\Delta\varphi)^6\rangle\,\langle(\Delta\varphi)^2\rangle - 420\,\langle(\Delta\varphi)^4\rangle\,\langle(\Delta\varphi)^2\rangle} - 1 \qquad = 0$$

Correlation functions
fields <-> phase

T. Schweigler et al. arXiv:1505.03126

correlation functions for the fields:

$$C(z_1,z_2) = \frac{\langle\Psi_1(z_1)\Psi_2^\dagger(z_1)\Psi_1^\dagger(z_2)\Psi_2(z_2)\rangle}{\langle|\Psi_1(z_1)|^2\rangle\langle|\Psi_2(z_2)|^2\rangle}$$

$$\boxed{C(z_1,z_2) \approx \langle\exp[i\varphi(z_1) - i\varphi(z_2)]\rangle}$$

$C(z_1,z_2)$ contains **all orders of connected** parts

$$C(z_1,z_2) = \exp\left[\sum_{k=1}^{\infty}(-1)^k \frac{\langle(\Delta\varphi)^{2k}\rangle_c}{(2k)!}\right]$$

for Gaussian fluctuations

$$C(z_1,z_2) = \exp\left[-\frac{1}{2}\,\langle(\Delta\varphi)\rangle^2\right]$$

Observable and non-gauss measure

T. Schweigler et al. arXiv:1505.03126

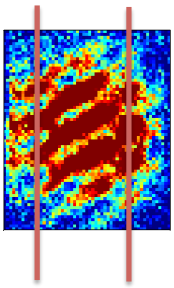
to study factorization of correlation functions we look at

$$C^{(2)}(z_1, z_2) = \langle[\varphi(z_1) - \varphi(z_2)]^2\rangle = \langle[\Delta\varphi(z_1,z_2)]^2\rangle$$

$$C^{(4)}(z_1, z_2, z_3, z_4) = \langle[\varphi(z_1) - \varphi(z_2)]^2[\varphi(z_3) - \varphi(z_4)]^2\rangle = \langle[\Delta\varphi(z_1,z_2)]^2[\Delta\varphi(z_3,z_4)]^2\rangle,$$

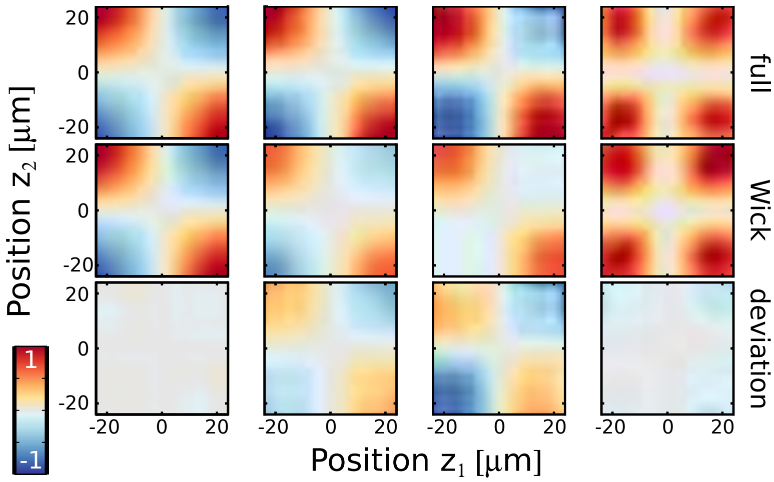
$\Delta\varphi(z_1, z_2) = \varphi(z_1) - \varphi(z_2)$

$\Delta\varphi$ *is* NOT restricted to 2π



$\Delta\varphi > 2\pi$

$C^{(4)}(z_1, z_2, -15, 15)$



experiment: T. Schweigler et al.
theory: V. Kasper, S. Erne

Characterising non-Gaussian phase fluctuations

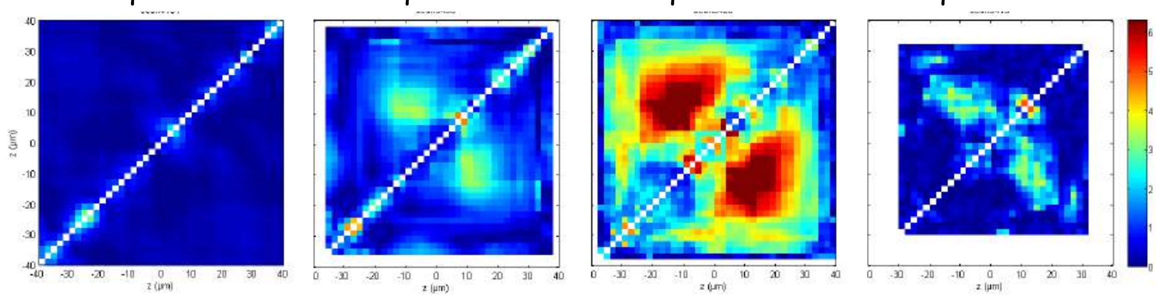
T. Schweigler et al. arXiv:1505.03126

Characterising the factorisation by the connected part: $\langle(\Delta\varphi)^4\rangle_c = \langle(\Delta\varphi)^4\rangle - 3\,\langle(\Delta\varphi)^2\rangle^2$

excess Kurtosis $$\gamma_2 = \frac{\langle(\Delta\varphi)^4\rangle}{3\,\langle(\Delta\varphi)^2\rangle^2} - 1$$

Experimental data, thermal state in a double well

plasma-freq. = 0 Hz	70 Hz	160 Hz	220 Hz
q = 0	q = 2.5	q = 3.4	q = 8.6

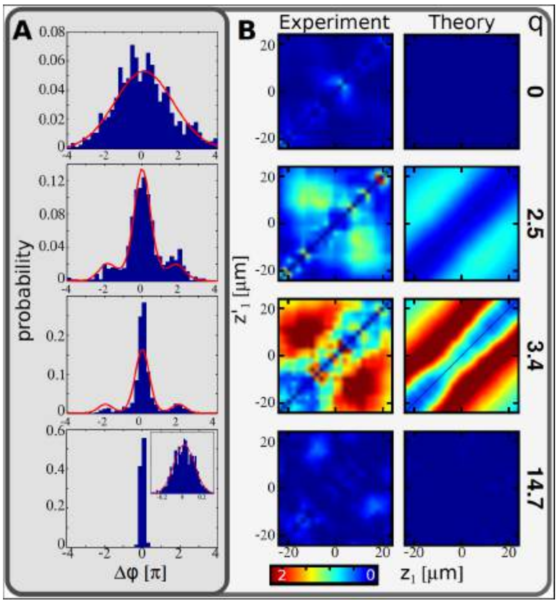


Quantifying factorization of correlation functions

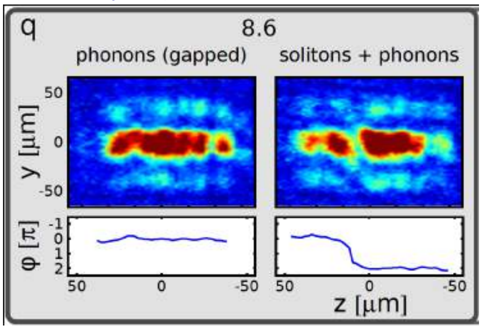
T. Schweigler et al. arXiv:1505.03126

full distribution function

Kurtosis



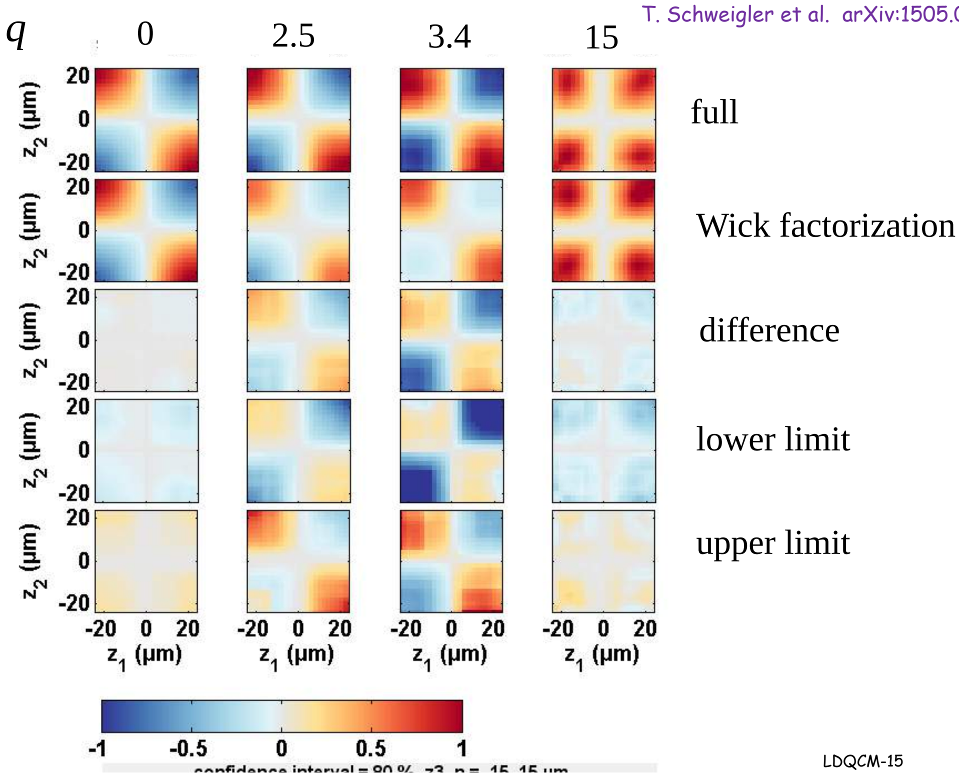
- the breakdown of factorization is evident in the **full distribution functions** of the phase by new peaks at multiples of 2π
- caused by the 2π ***periodic*** SG Hamiltonian -> 2π phase jumps, 'kinks' = SG solitons



- ***SG Solitons*** are topological excitations
- Phase fluctuations around *topologically different Vaccua*

4-point phase correlators

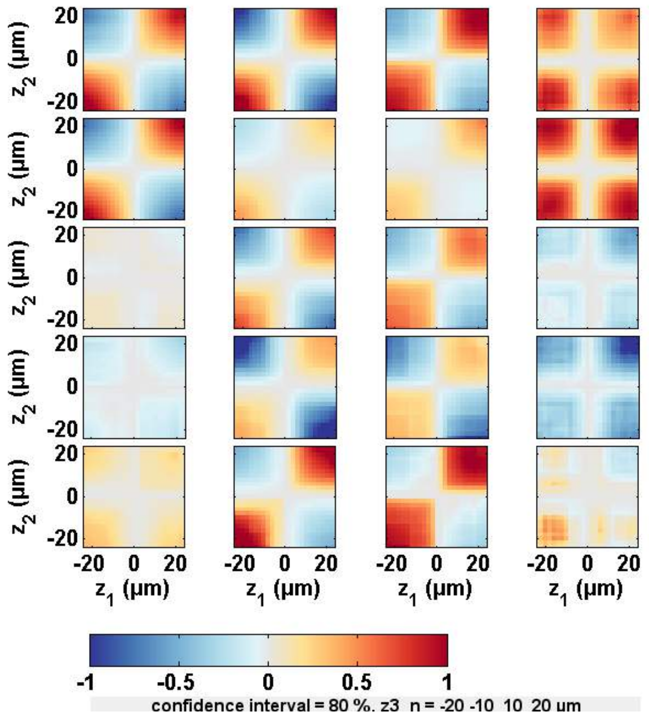
T. Schweigler et al. arXiv:1505.03126



6-point phase correlators

T. Schweigler et al. arXiv:1505.03126

q 0   2.5   3.4   15



full

Wick factorization

difference

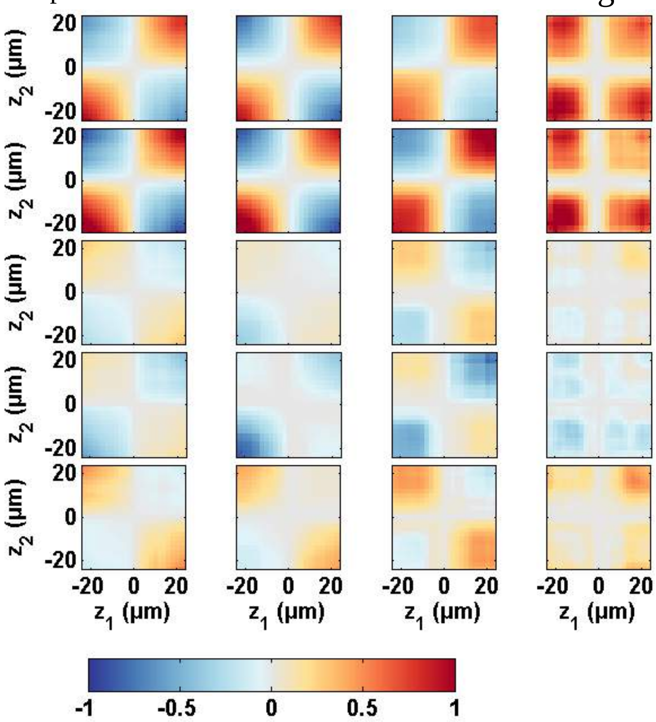
lower limit

upper limit

6-point phase correlators, connected part

T. Schweigler et al. arXiv:1505.03126

ω_p 0 Hz   70 Hz   160 Hz   big



full

disconnected part

connected part

lower limit

upper limit

Remove Solitons

Strongly coupled $q = 8.6$ $\omega_p > 500$ Hz

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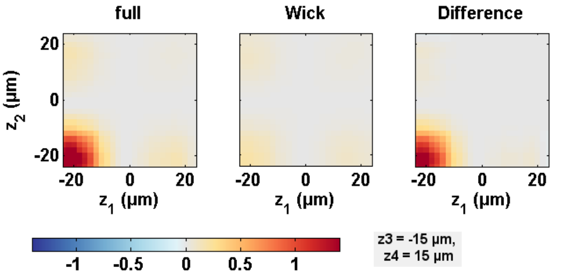
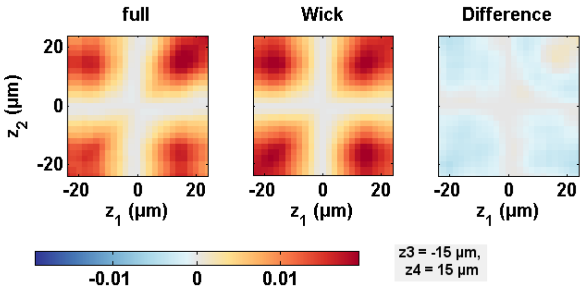
4-point correlator does not factorize:



without Solitons:



Remove Solitons

intermediate coupling $q = 3.4$ $\omega_p = 160$ Hz

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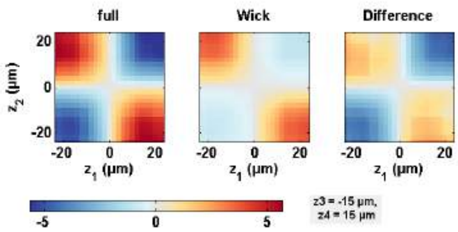
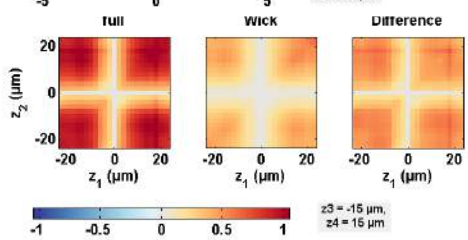
4-point correlator does not factorize:

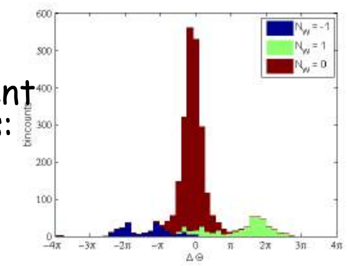


without Solitons:

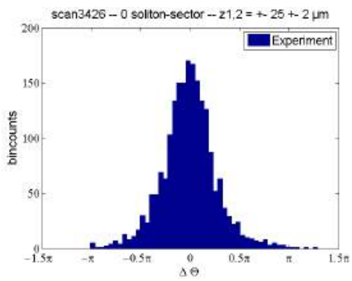


phase distribution:

different sectors:



without solitons:



What have we learned

T. Schweigler et al. arXiv:1505.03126

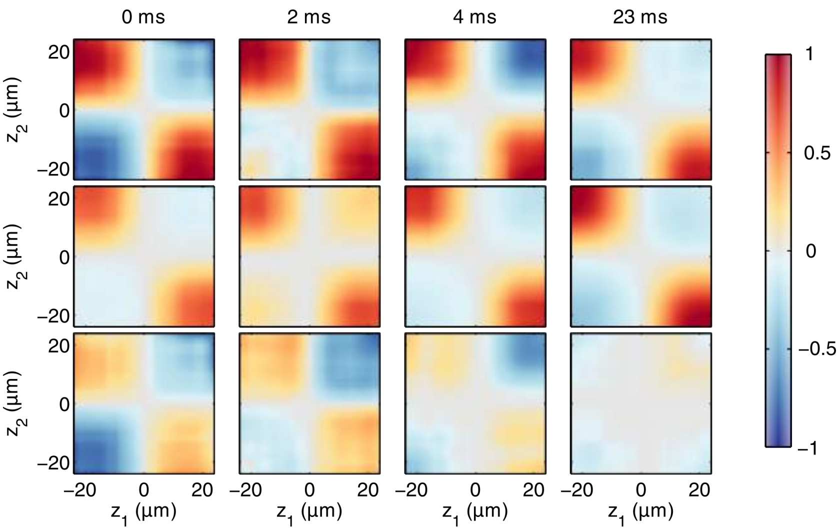
- **high order** (>10) **correlation** functions are accessible in experiment
- **full distribution functions** and the **connected part** of the higher order correlation functions contain genuine information about the quantum field theory
  - quasi particles
  - interaction of quasi particles
  - vacuum states
- gives insight in the ***effective theories*** describing the many body system
  - for our resolution 6^{th} order is sufficient
    -> necessary to take perturbation expansion up to 3^{rd} order (3-3 scattering)

Quench from J>0 to J=0

very preliminary

experiment: T. Schweigler et al.

Initial state q=3,4: non-Gaussian, dynamics Gaussian

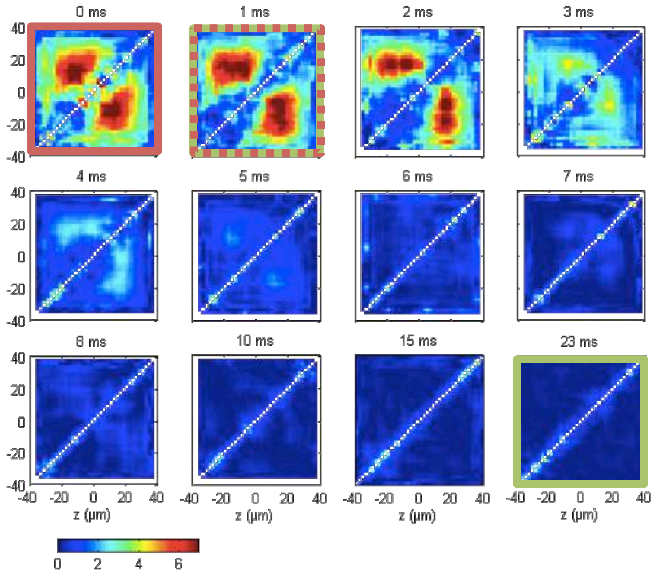


collaboration with Berges & Gasenzer groups, Heidelberg

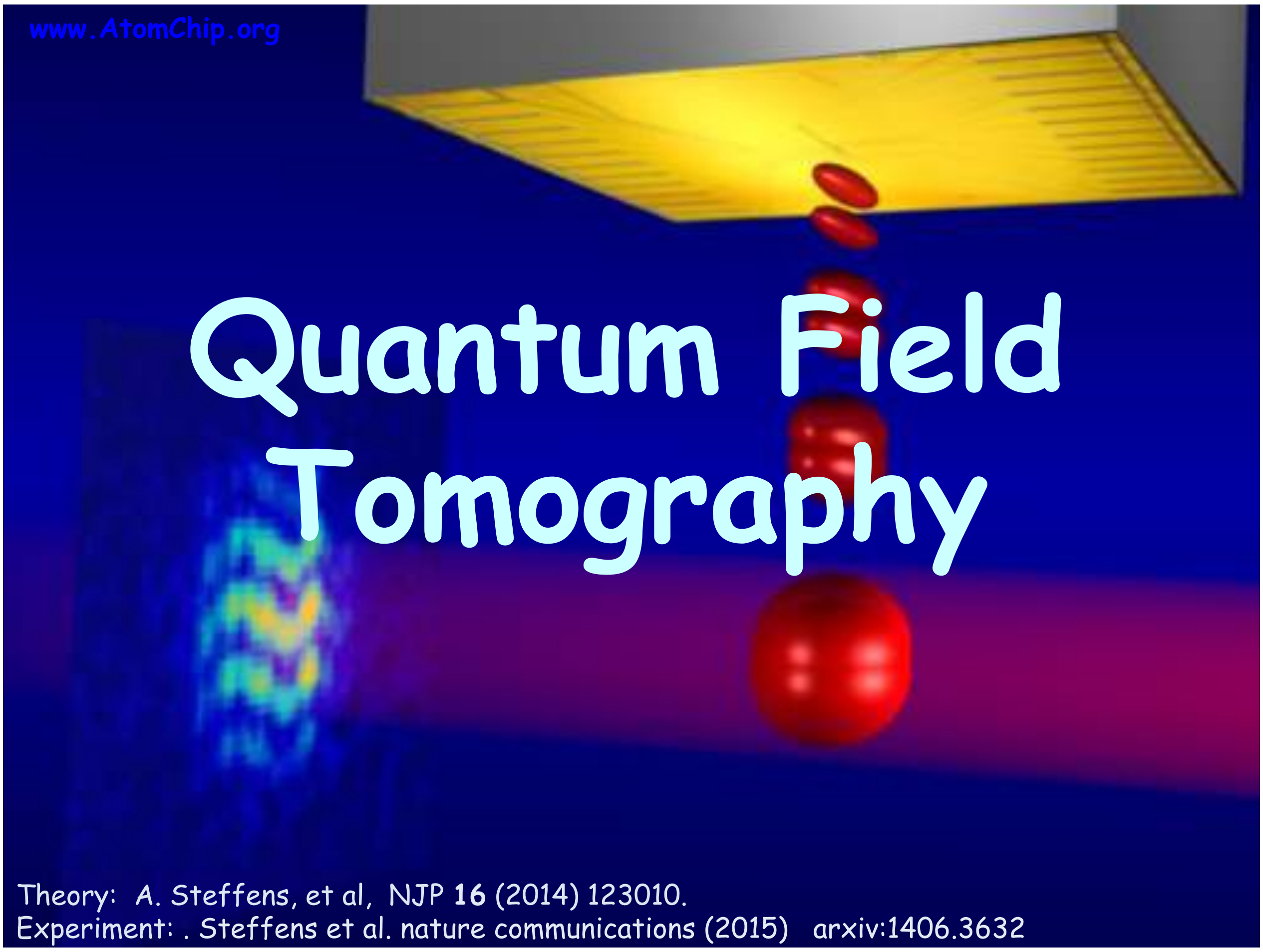
Quench from J>0 to J=0

experiment: T. Schweigler et al.

Initial state q=3,4: non-Gaussian, dynamics Gaussian



collaboration with Berges & Gasenzer groups, Heidelberg



Reconstructing Quantum States

- Reconstruction of an unknown state based on data alone
- Generically, need d^2 expectation values to reconstruct an unknown state in d dimensions
- full tomography tools for state identification inefficient, especially for continuous systems
- Brought down to $O(rd \log^2(d))$ for approx low-rank state with compressed sensing.
- Applicable for medium sized systems in conjunction with **model selection**. Approaches are based on using the right "data set" with the appropriate "sparsity structure" to capture quantum many-body systems.
  - over permutation-invariant tomography
  - matrix-product state tomography
  - ....

Quantum Field Tomography

efficient tomographic state reconstruction for continuous quantum fields

continuous Matrix Product States (cMPS) naturally incorporate the locality present in realistic physical settings of locally interacting quantum field

Phase correlation functions

$$C^{(n)}(x_1, \ldots, x_n) = \mathrm{Re}\left\langle \mathrm{e}^{\mathrm{i}(\hat\theta_{x_1} - \hat\theta_{x_2} + \hat\theta_{x_3} - \cdots + \hat\theta_{x_{n-1}} - \hat\theta_{x_n})} \right\rangle$$

with $$\hat\psi^\dagger(x) = \hat n(x)^{\frac{1}{2}} e^{\hat\theta_x}$$

Extract low-order correlation functions

$$C^{(n)}(\tau_1, \ldots, \tau_{n-1}) = \sum_{\{k_j\}=1}^{d^2} \rho_{k_1,\ldots,k_{n-1}} e^{\lambda_{k_1}} \ldots e^{\lambda_{k_{n-1}}\tau_{n-1}}$$

Reconstruct ***continuous matrix product states***

$$|\psi_{Q,R}\rangle = \mathrm{tr}_{\mathrm{aux}}\left(\mathcal{P} e^{\int_0^L dx (Q\otimes 1 + R \otimes \Psi^\dagger(x))}\right)|0\rangle$$

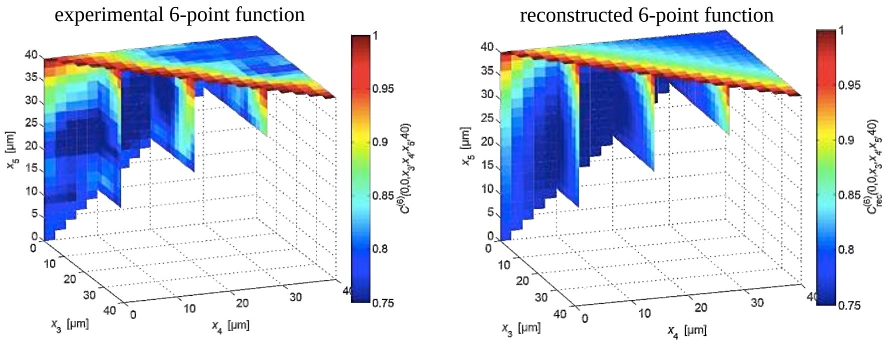
Methods: Matrix pencils, prony methods

Steffens, Friesdorf, Langen, Rauer, Schweigler, Huebener, Schmiedmayer, Riofrio, Eisert, Nature Comm (2015)
Steffens, Riofrio, Huebener, Eisert, New J Phys **16**, 123010 (2014)

Quantum field tomography

A. Steffens et al. Nature Comm (2015) arxiv:1406.3632

Use 2- and 4-point functions to reconstruct higher-order functions via continuous matrix product states with bond length 2



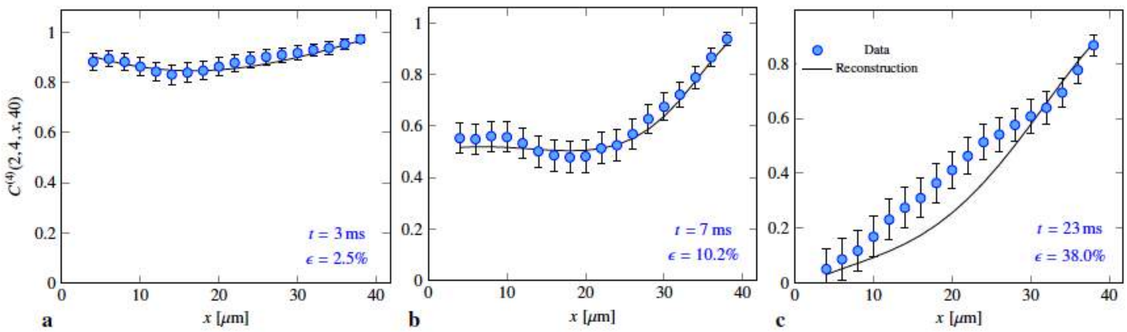
reconstruction of a quantum field with very weak assumptions

Theory:
A. Steffens, C. Riofrıo, R. Hubener, and J. Eisert, "Quantum field tomography," NJP **16** (2014) 123010.

Quantum field tomography

A. Steffens et al. Nature Comm (2015) arxiv:1406.3632

Use 2- and 4-point functions to reconstruct higher-order functions via continuous matrix product states with bond length 2

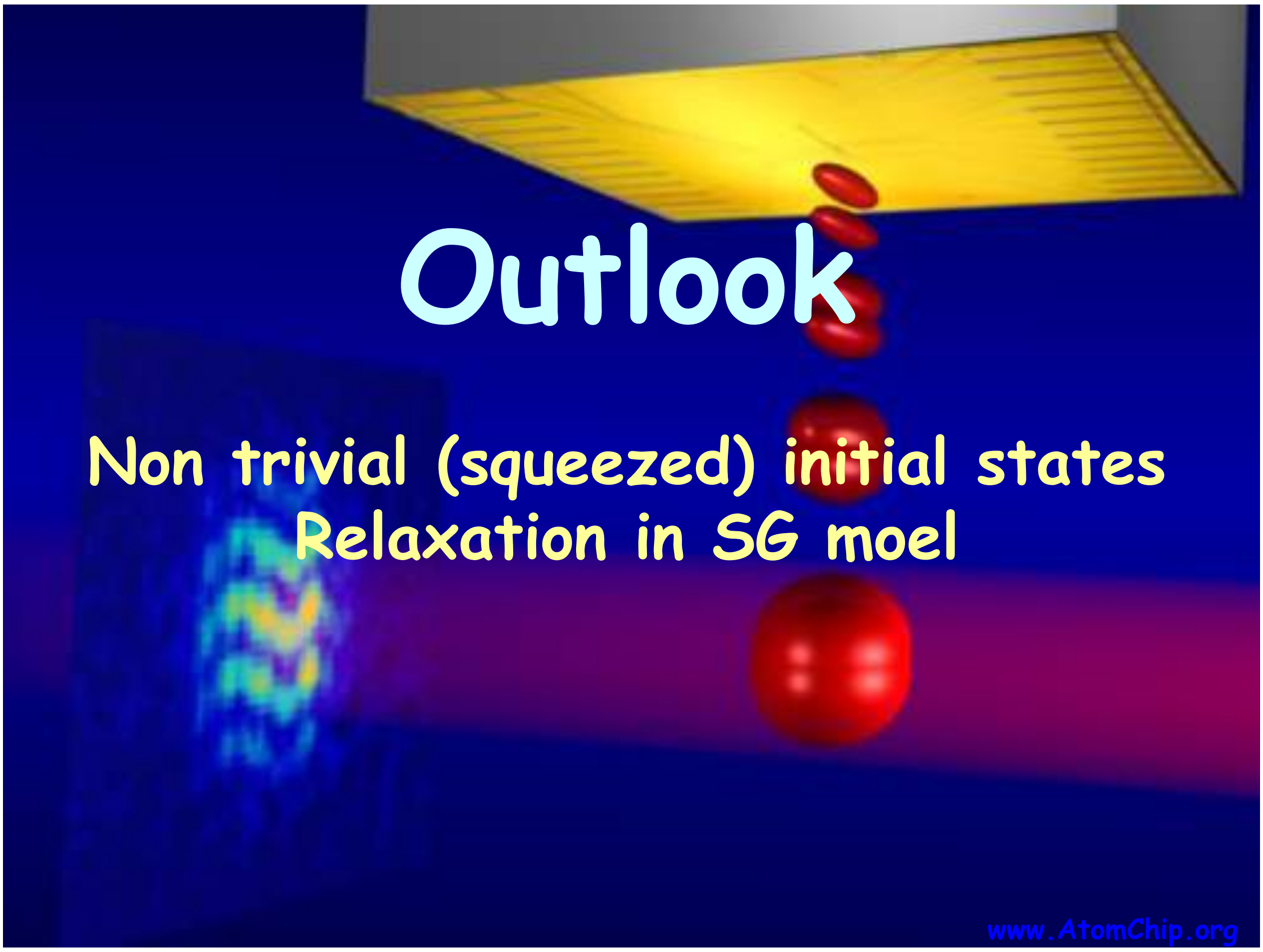


reconstruction of the C-MPS wave functions gets worse with time

C-MPS with bond length 2 have finite entanglement

Question: Can one build a measure for entanglement growth after the quench?

similar to data compressibility criteria?



Optimal Control of Splitting
fast squeezing in a multi mode system

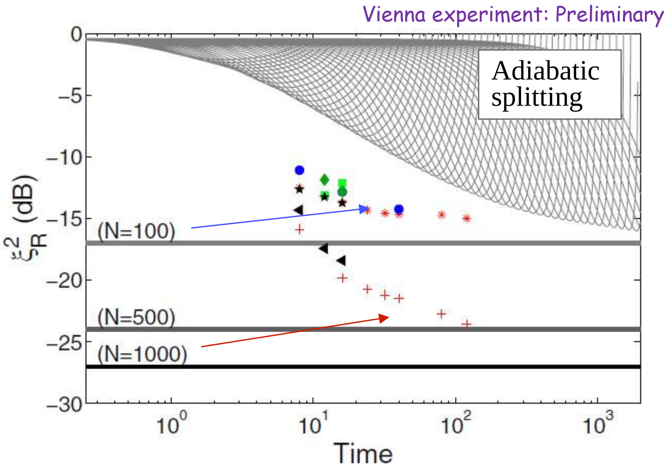
Vienna Center for Quantum Science and Technology

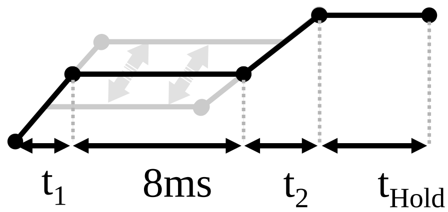
Vienna experiment: Preliminary

Optimal Controll applied to the problem of the fluctuation properties in splitting a BEC

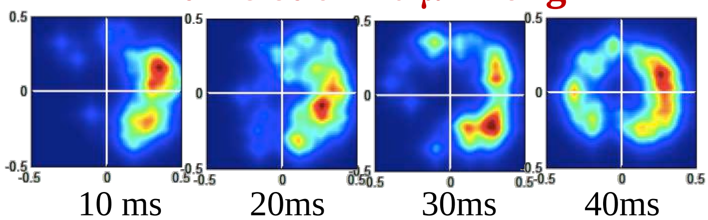
J. Grond et al. PRA **79**, 021603 R (2009)
J. Grond et al. PRA **80**, 053625 (2009)

- Fancy splitting ramps inspired by OCT: t_1+t_2 = 17ms
- Leads to dramatic change of statistical distribution of interference





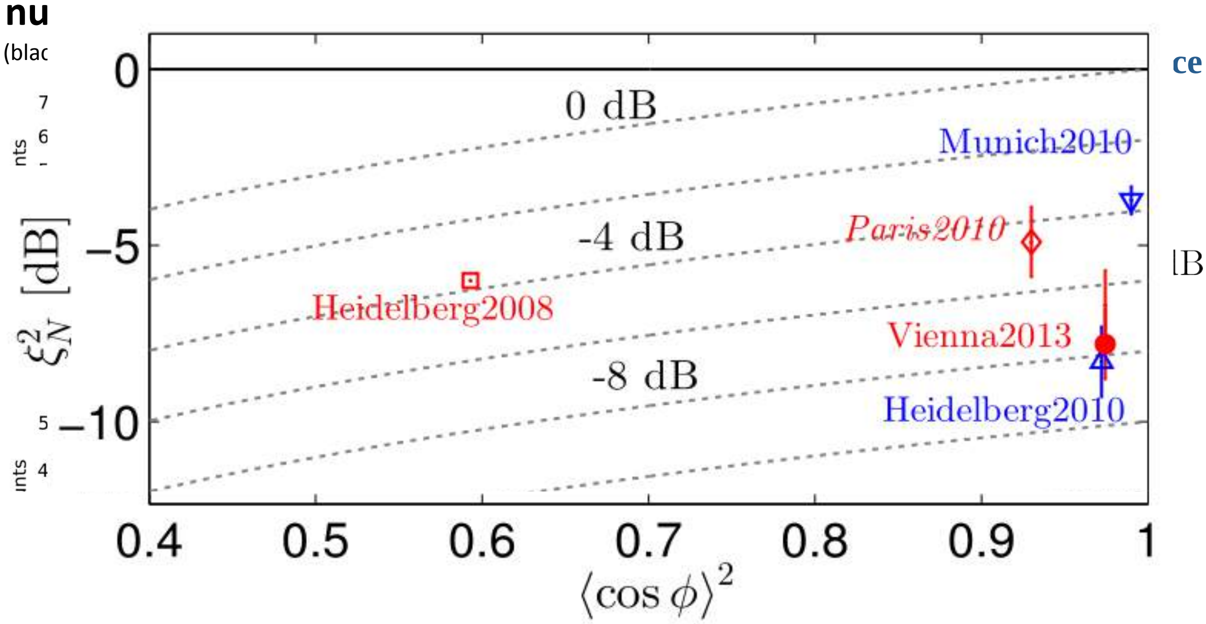
full cloud 140 μm long

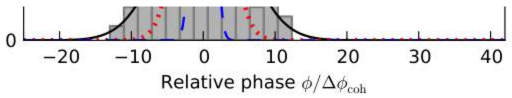


Squeezing

$N = 1200$ atoms, $\mu \simeq 0.5\,\text{kHz}$, $T \simeq 25\,\text{nK}$ $(0.5\ \text{kHz})$

T. Berrada, et al., Nat. Comm **4**, 2077 (2013)





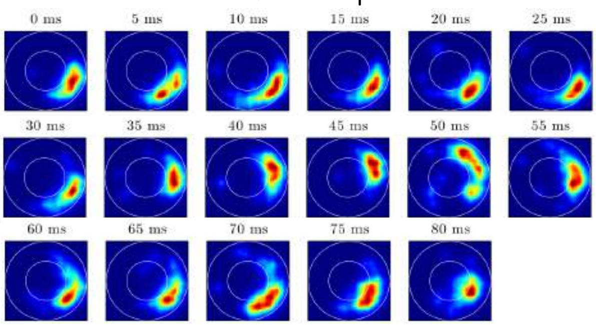
when correcting for measurement noise:

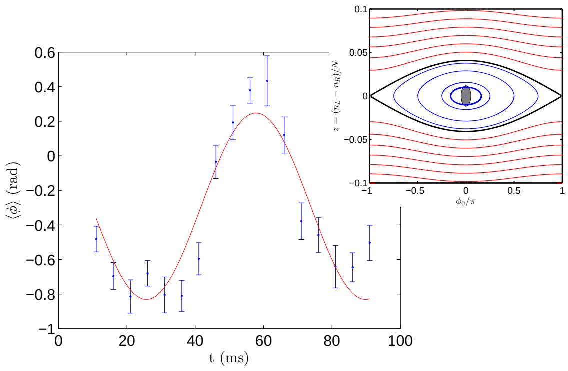
$$\Delta n\, \Delta\phi \sim 1$$

Evolution of ξ^2~-8dB 1d gas

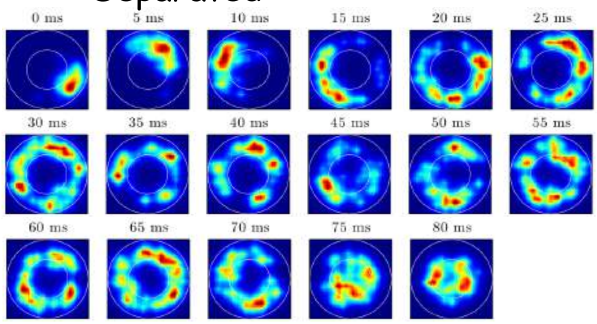
T. Berrada prelininary

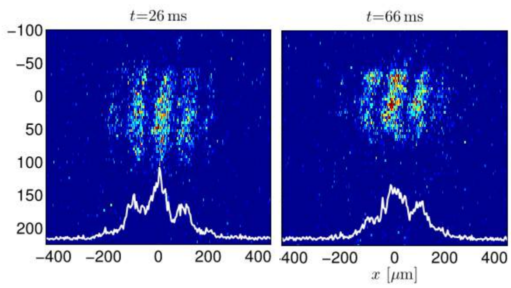
Tunnel Coupled ω_p=14Hz





Separated

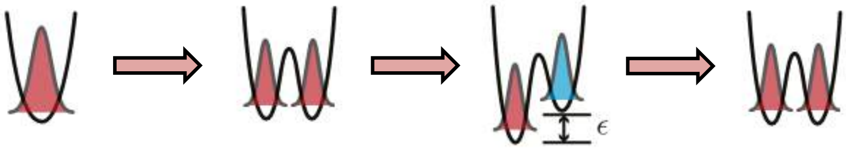


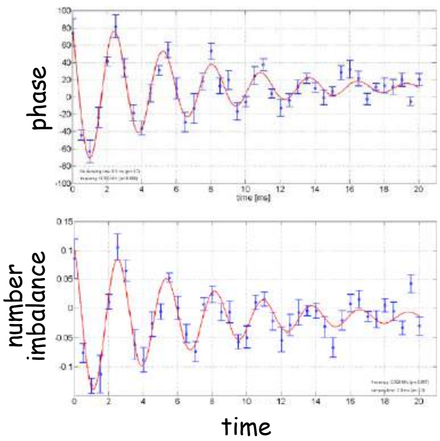


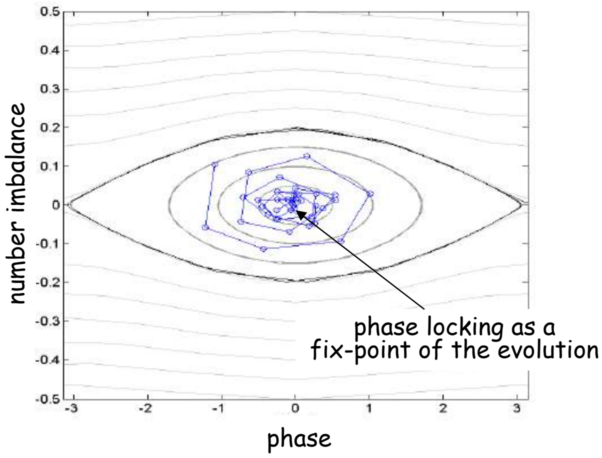
Relaxation in coupled superfluids

re-coupling starts SG model with a specific phase
-> study phase locking

experiment: M. Pigeur
theory: E. DelaTorre, E. Demler

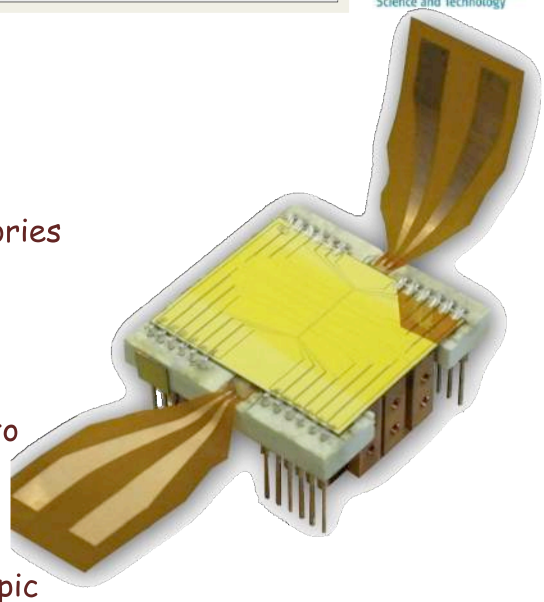






What have we learned

- **high order** (>10) **correlation** functions are accessible in experiment
- **Higher order correlation functions** and the question if they factorize (**full distribution functions**) gives insight in the effective theories describing the many body system
  - quasi particles
  - interaction of quasi particles
  - vacuum states
- **Quantum field tomography** opens up a way to extract information by using model cMPS wave-functions
- Experiments allow to probe how classical statistical properties emerge from microscopic quantum evolution through de-phasing of many body eigenstates.



Schweigler et al. arXiv:1505.03126

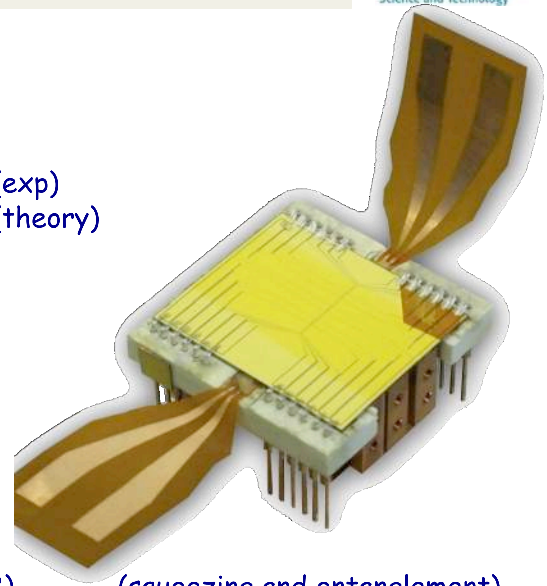
Steffens, et al., Nature Comm (2015)
Steffens, et al., NJP **16**, 123010 (2014)

Papers

Probing Quantum Fields by correlations
Schweigler et al. arXiv:1505.03126

Quantum Field Tomography
Steffens, et al., Nature Comm. (2015) (exp)
Steffens, et al., NJP **16**, 123010 (2014) (theory)

Non equilibrium an relaxation in 1d systems
Gring et al., Science **337**, 1318 (2012)
Kuhnert et al., PRL **110**, 090405 (2013)
Smith et al. NJP **15**, 075011 (2013)
Langen et al., Nature Physics **9**, 460 (2013)
Geiger et al. NJP **16** 053034 (2014)
Langen et al. Science **348**, 207 (2015)



Interferometer with trapped BEC
Berrada, et al., Nature Comm. **4**, 2077 (2013) (squeezing and entanglement)
Van Frank, et al., Nature Comm. **5**, 4009 (2014)

Coolig a 1d quantum gas
Rauer et al., arXiv:1505.04747

Atom Chip Experiment

S. Manz, T. Betz, R. Bücker, **T. Berrada**, S. vanFrank,
M. Pigeur, A. Perrin, T Schumm, JF Schaff, R. Wu,
M. Bonneau

M. Kuhnert, M. Gring, **B. Rauer, Th. Schweigler**
D. Smith, R. Geiger, **T. Langen**

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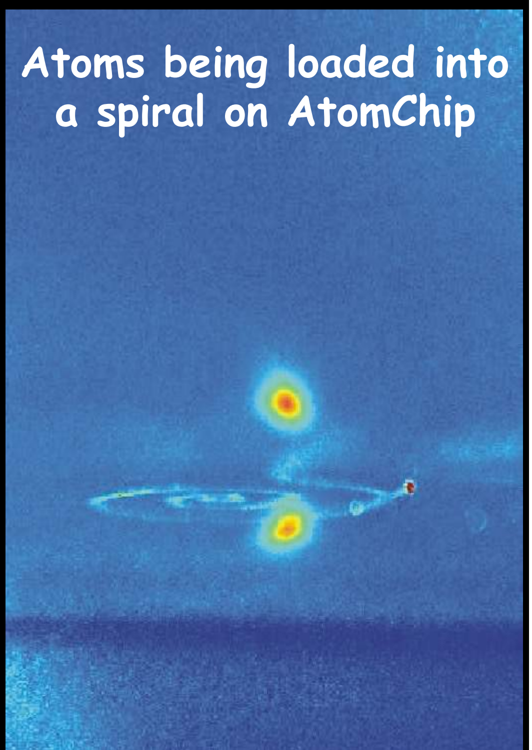
E. Demler, T. Kitagawa + … (Harvard)
T. Gasenzer, J. Berges, **S. Erne, V. Kasper** + … (Heidelberg)
T. Calarco, S. Montanegro + … (Univ. Ulm)
J. Eisert + …. (FU-Berlin)

EU: SIQS, QIBEC, AQuS
AT: FWF, CoQuS, Wittgenstein, Stadt Wien
ERC AdG: QuantumRelax



www.AtomChip.org