# Lattice models with explicit, hidden and emergent supersymmetry

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# Introduction

- Supersymmetry can play a role in condensed matter systems as an explicit, hidden or emergent symmetry
  - Models with explicit and hidden susy:
    interplay between susy and bethe ansatz

**Hagendorf, Fokkema, Huijse, J Phys A 2014**

  - Models with emergent susy:
    interplay between susy and RG

**Huijse, Bauer, Berg, PRL 2015**

# Supersymmetric quantum mechanics

- Hamiltonian is part of superalgebra:

$$Q^2 = 0,\ (Q^\dagger)^2 = 0,\ H = \{Q, Q^\dagger\},\ [\,Q^{(\dagger)}, F\,] = (-)Q^{(\dagger)}$$

$$[\,H, F\,] = [\,H, Q\,] = [\,H, Q^\dagger\,] = 0,$$

$Q:$ Supercharge, $H:$ Hamiltonian, $F:$ Fermion number

- Consequences:
  - $E \geq 0$
  - Zero energy states are annihilated by Q and $Q^\dagger$
  - Positive energy states come in pairs (superpartners): 2 states with same energy and differing in fermion number by one

# Lattice models with explicit supersymmetry

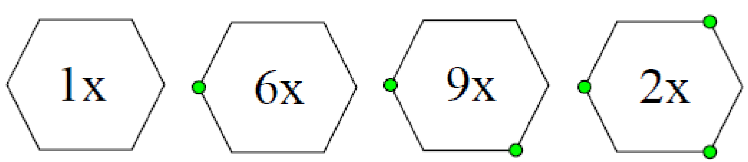
- Supercharge:

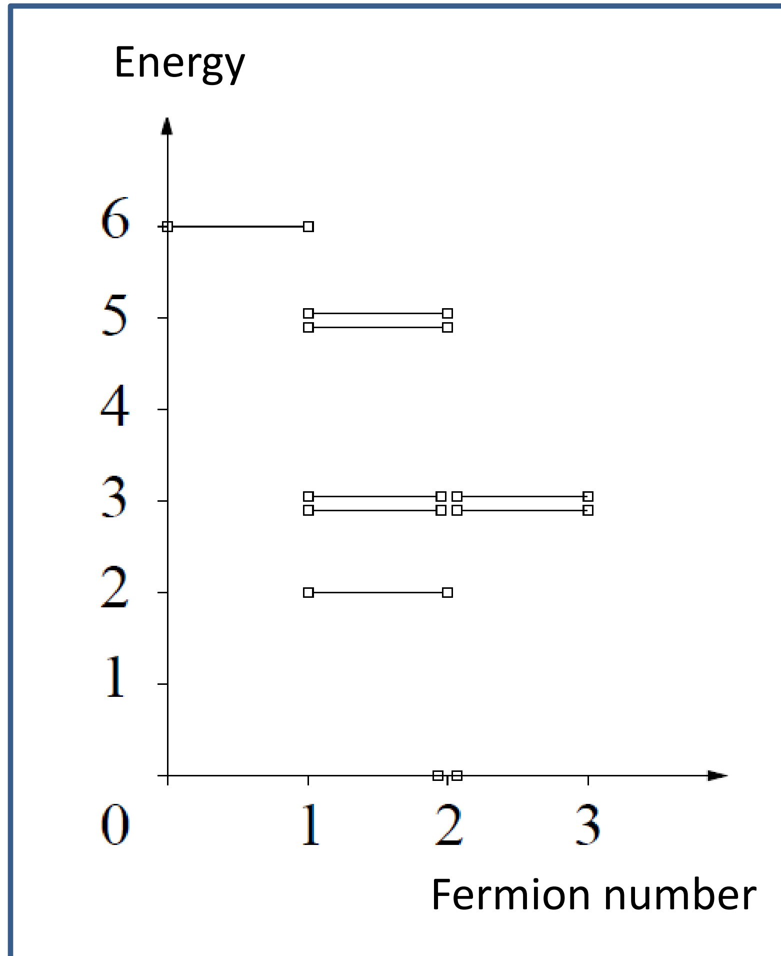
$$Q^\dagger = \sum_i c_i^\dagger P_{\langle i \rangle} = \sum_i c_i^\dagger \prod_{j \text{ next to } i} (1 - n_j)$$

- Hamiltonian:

$$H = \sum_{\langle i,j \rangle} P_{\langle i \rangle} c_i c_j^\dagger P_{\langle j \rangle} + \sum_i P_{\langle i \rangle}$$

- Example: 6-site periodic chain

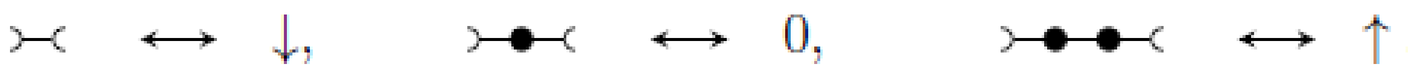



Fendley, Schoutens, de Boer, PRL (2003)

# Lattice models with explicit supersymmetry in 1D

- $M_k$ models with k-clustering constraint:
  on a chain *at most* k consecutive particles are allowed
- Supercharge, Q, removes the n-th particle of m consecutive particles with amplitude $\lambda_{m,n}$
- Nilpotency of Q leaves k-1 free parameters
- There exists a mapping to spin-k/2 models. Example for k=2:

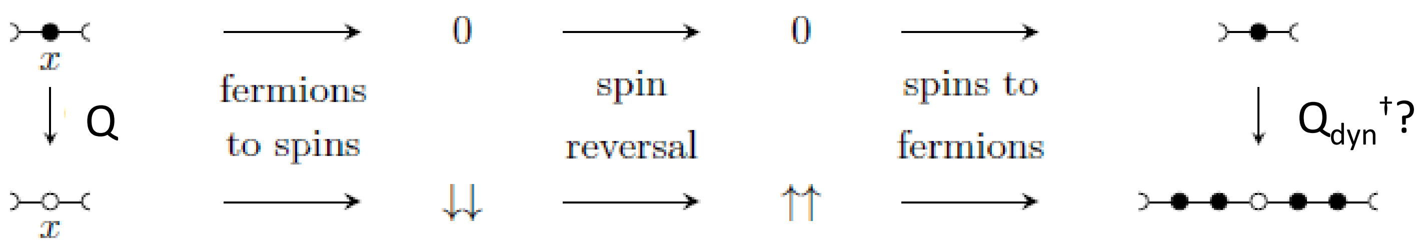


- Spin reversal symmetry fixes all parameters uniquely
- Continuum theory: k-th superconformal minimal model with N=(2,2) susy

Fendley, Schoutens, de Boer, PRL (2003)
Fendley, Nienhuis, Schoutens, JPhysA (2003)
Hagendorf, Fokkema, LH, JPhysA (2014)
Hagendorf, LH (in preparation)

# Hidden dynamical supersymmetry in 1D lattice models

- Spin reversal suggests hidden susy. Example:

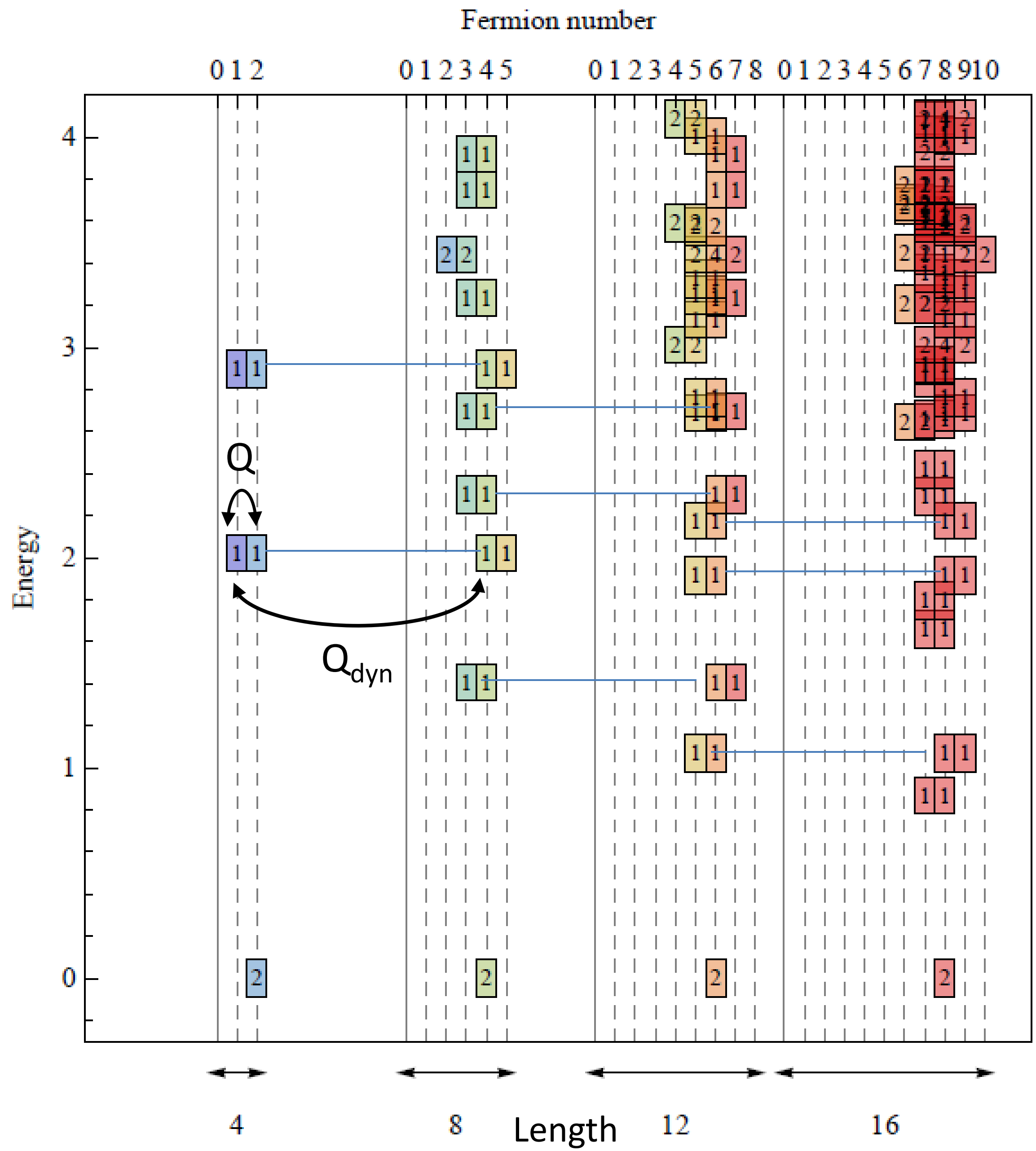



- $M_k$ models indeed have hidden susy $H = \{Q^{\dagger}_{\mathrm{dyn}}, Q_{\mathrm{dyn}}\}$
  – For certain momentum sectors ($T^{k+2} = 1$)
  – For parameters that preserve spin reversal symmetry
- Dynamical supercharge changes the lattice and is therefore hidden $V_{L,f} \overset{Q^{\dagger}_{\mathrm{dyn}}}{\rightarrow} V_{L+k+2,f+k+1}$

$V_{L,f}$ : Fock space spanned by configurations of f particles with k-neighbor exclusion on L site chain

Hagendorf, Fokkema, LH, JPhysA (2014); Hagendorf, LH (in preparation); Hagendorf (2013); Meidinger, Mitev (2013)

# Hidden susy $M_2$ model




Hagendorf, Fokkema, LH, JPhysA (2014)

# $M_k$ models with spatial modulation

- Site dependent parameters: Supercharge, Q, removes the n-th particle of m consecutive particles at site x with amplitude $\lambda_{m,n,x}$
- Hidden susy requires periodicity:

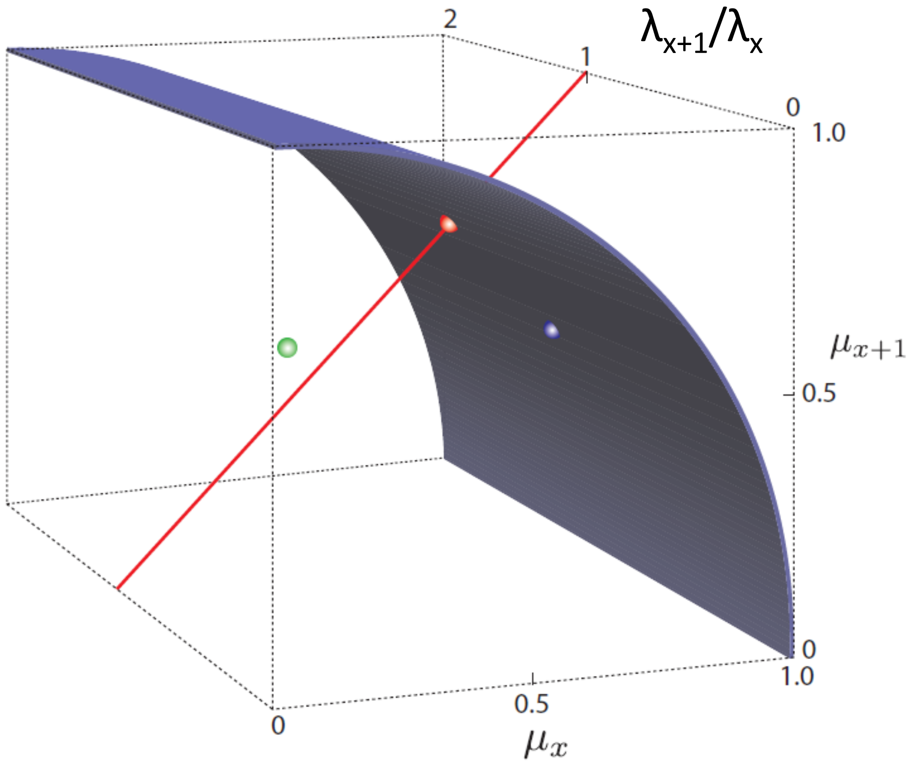
$$\lambda_{m,n,x+p} = \lambda_{m,n,x},$$

$$p = \begin{cases} k+2 & k \text{ odd} \\ k/2+1 & k \text{ even} \end{cases}$$

- Submanifold with hidden susy coincides with submanifold for which we find a Bethe Ansatz solution!

$M_2$ model: BA+hidden susy manifold

$$\lambda_{1,1,x} = \lambda_x, \quad \lambda_{2,1,x} = \lambda_x \mu_x, \quad \lambda_{2,2,x} = \lambda_x \mu_{x-1}$$




Hagendorf, Fokkema, LH, JPhysA (2014)

Hagendorf, LH (in preparation)

Nienhuis, Blom (2012); Fokkema, Schoutens (2015)

# $M_k$ models with spatial modulation: Bethe Ansatz susy interplay

- Coordinate Bethe Ansatz: wavefunction is linear superposition of products of single particle wavefunctions
- Ansatz -> Bethe equations for quasimomenta of the particles
- Solution: quasimom $(u_1, .., u_f)$ -> energy and wavefunction
- Explicit susy: $Q^\dagger$: $(N,f;u_1, .., u_f)$ -> $(N,f+1;u_1, .., u_f, u_{f+1}=0)$
- Hidden + explicit susy: ${Q_{dyn}}^\dagger Q^\dagger$ adds exact string centered around zero, e.g. in $M_2$ model:

$$(u_{f+1}, u_{f+2}, u_{f+3}, u_{f+4})=u_0+(0,\pi/4, \pi/2, 3\pi/4) \text{ with } u_0=0$$

- $M_2$ model: zero energy state is condensate with all quasimomenta equal to zero.

Hagendorf, Fokkema, LH, JPhysA (2014)
Hagendorf, LH (in preparation)

# LATTICE MODELS WITH EMERGENT SUSY

# Lattice models with emergent supersymmetry

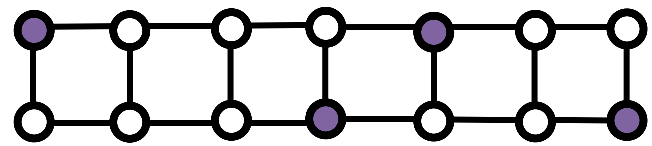
- Emergent N=1 susy:
  - Tricritical Ising model [Friedan, Qiu, Shenker]
  - Josephson Junction arrays [Foda]
  - Majorana fermions at boundary of topological superconductor coupled to Ising order parameter [Grover, Vishwanath, Sheng]
  - Interacting majorana fermions [Rahmani, Zhu, Franz, Affleck]
  - Coupled wires [LH, Bauer, Berg; Sitte et al.]
- Emergent N=2 susy:
  - Spin-1/2 XXZ chain with anisotropy $\Delta$=1/2 (also hidden!) [Fendley, Nienhuis, Schoutens; Veneziano, Wosiek]
  - Models with Ising-KT multicritical point (e.g. spinless fermions on a ladder) [LH, Bauer, Berg]

# Lattice models with $Z_2$ and U(1) symmetry

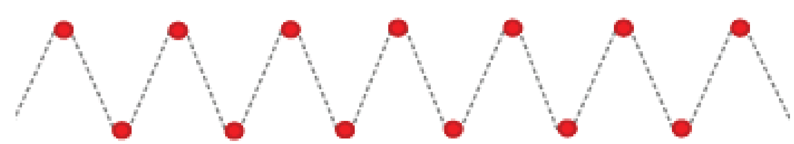
- Examples:
  - Spin-1 chain with SU(2) broken down to U(1)x$Z_2$

↑↓↓0↑0↑↑↓00↓0

  - Spinless fermions on a squareladder



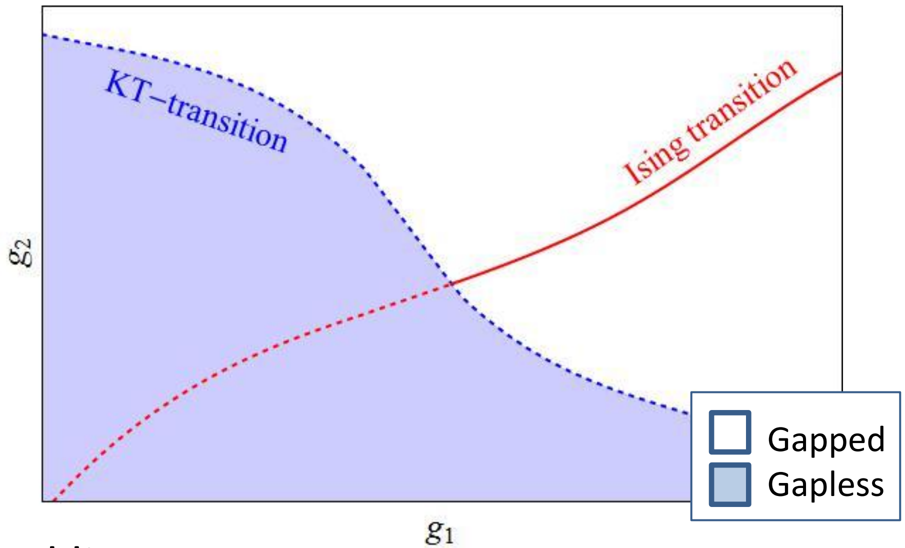
  - polar molecules in 1D optical lattice (zig-zag instability)



**[Fendley, Nienhuis, Schoutens 2003; Bauer, LH, Berg, Troyer, Schoutens 2013; Ruhman, Dalla Torre, Huber, Altman 2012]**

# Lattice models with $Z_2$ and U(1) symmetry

- Z2 ↔ Ising transition: c=1/2 majorana fermion (*velocity u*)
- U(1) ↔ quasi long range order and KT transition: c=1 boson (*v*)
- Multicriticality: c=3/2, **if u=v and no coupling**
  - N=1 susy in gapless phase at Ising critical line
  - N=3 susy at Ising-KT multicritical point
- Marginal coupling ($\lambda$): boson density couples to fermion mass
- Susy was key in obtaining results for coupled theories




| | λ=0 | λ≠0 |
|---|---|---|
| **u<v** | No susy | ***N=1* (+N=2) *susy and slow flow*** |
| **u=v** | N=1(+N=2) susy | |
| **u>v** | No susy | ? |

LH, Bauer, Berg, PRL (2015)
Sitte, Rosch, Meyer, Matveev, Garst, PRL (2009)

# Physics at and near multicriticality

**u ≤ v**

| | **λ=0** | | **λ≠0** | |
|---|---|---|---|---|
| | Critical line in gapless phase | Multicritical point | Critical line in gapless phase (N=1 susy) | Multicritical point (N=2 susy) |
| Physical velocities | No flow | | $u(\ell) \sim v(\ell) \sim e^{-a_1 \ell^{1/5}} \to 0$ | $u(\ell) \sim v(\ell) \sim \ell^{-1/4} \to 0$ |
| Specific heat coefficient | Finite | | $e^{a_2(-\ln T)^{1/5}} \to \infty$ | $(-\ln T)^{1/4} \to \infty$ |
| Ising/ fermionic gap | $\Delta_m \sim \lvert m \rvert$ | | $\lvert m \rvert e^{-a_3(-\ln \lvert m \rvert)^{-1/5}}$ | $\lvert m \rvert (-\ln \lvert m \rvert)^{-1/4}$ |
| KT/bosonic gap | n/a | $\Delta_\delta \sim e^{c_0/\sqrt{\delta}}$ | n/a | $\frac{1}{\sqrt{\ln(\lambda_0/\delta)}} e^{-\frac{1}{4}(\ln(\lambda_0/\delta))^2}$ |

$\ell$ : RG scale, $\delta = g_0 - g^*$ : distance to KT critical point

LH, Bauer, Berg, PRL (2015)
Sitte, Rosch, Meyer, Matveev, Garst, PRL (2009)

# Conclusions

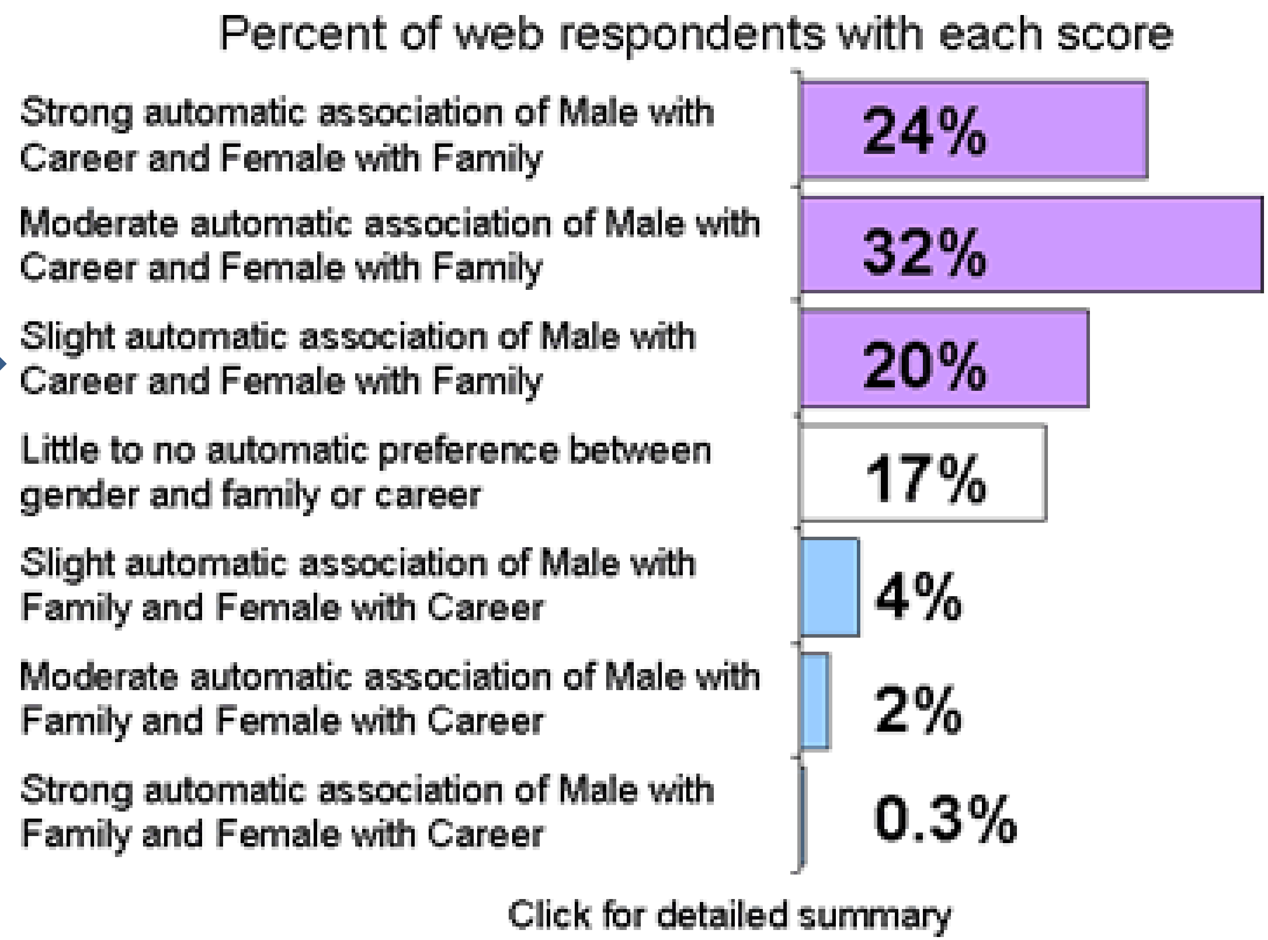
- Lattice models can exhibit supersymmetry as an explicit, hidden or emergent symmetry
- It serves as a powerful tool to obtain exact/analytic results, directly (Witten index + cohomology) or indirectly through enhanced symmetry (BA + RG)
- Supersymmetry is an emergent property of various condensed matter systems that can be realized in the lab

# AND NOW FOR SOMETHING DIFFERENT…

Note added for online version: I took the fact that I was the only female speaker at the conference as an opportunity to discuss subtle gender bias and things we can do to help improve diversity in the physics community.

# Test your subtle gender bias at: implicit.harvard.edu

My result



# Subtle gender bias

Science faculty's subtle gender biases favor male students –
C. Moss-Racusin, et al. PNAS (2012)
From the abstract:
"In a randomized double-blind study (n=127), science faculty from research-intensive universities rated the application materials of a student who was randomly assigned either a male or female name for a laboratory manager position. **Faculty participants rated the male applicant as significantly more competent and hireable than the (identical) female applicant.** These participants also selected a higher starting salary and offered more career mentoring to the male applicant. **The gender of the faculty participants did not affect responses**, such that female and male faculty were equally likely to exhibit bias against the female student."

# Predetermining criteria makes us less prone to subtle bias

Constructed Criteria - Redefining Merit to Justify Discrimination
Uhlmann and Cohen, American Psychological Society (2005)

ABSTRACT— This article presents an account of job discrimination according to which people redefine merit in a manner congenial to the idiosyncratic credentials of individual applicants from desired groups. In three studies, participants assigned male and female applicants to gender-stereotypical jobs. However, they did not view male and female applicants as having different strengths and weaknesses. Instead, **they redefined the criteria for success at the job as requiring the specific credentials that a candidate of the desired gender happened to have. Commitment to hiring criteria prior to disclosure of the applicant's gender eliminated discrimination**, suggesting that bias in the construction of hiring criteria plays a causal role in discrimination.

# What helps to improve diversity?

- Get more role models
- Determine criteria before starting the search
- Raise awareness: give search committees literature on subtle gender bias