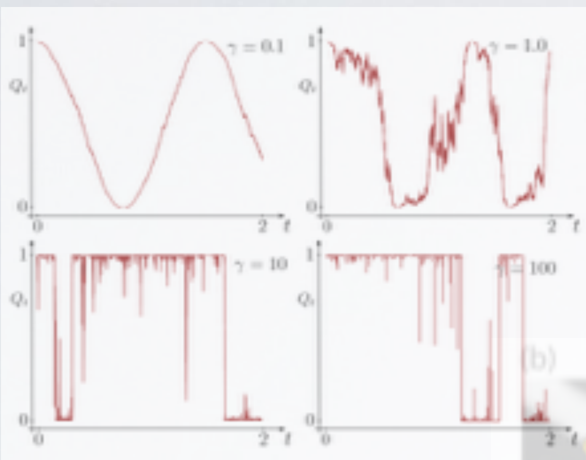
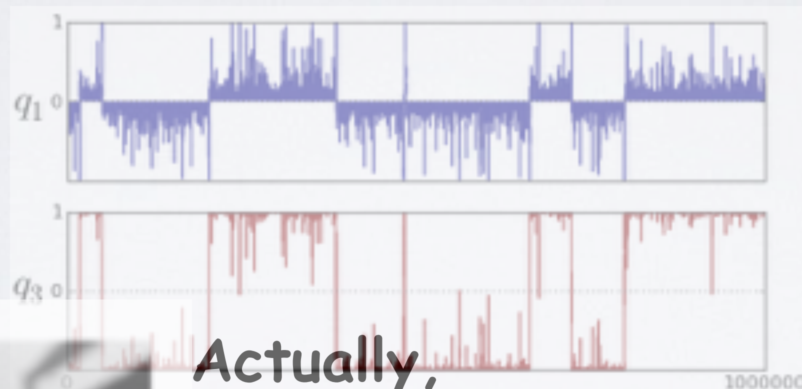
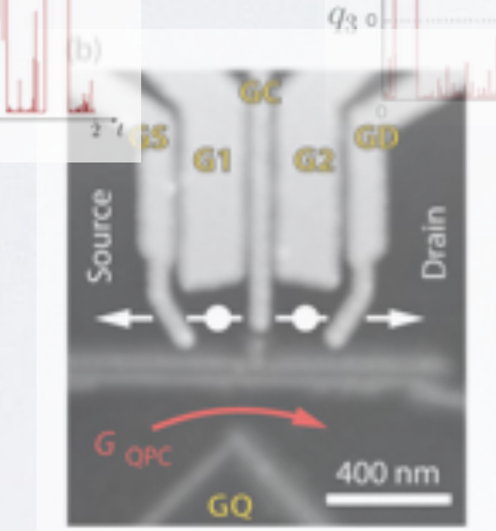
Statistical Aspects of Quantum State Monitoring and Applications







Actually,
statistics of « quantum trajectories »,
statistics of quantum jumps and spikes,
in system monitoring.
Application:
control and a simple Maxwell demon.

D.B., with M. Bauer and A. Tilloy

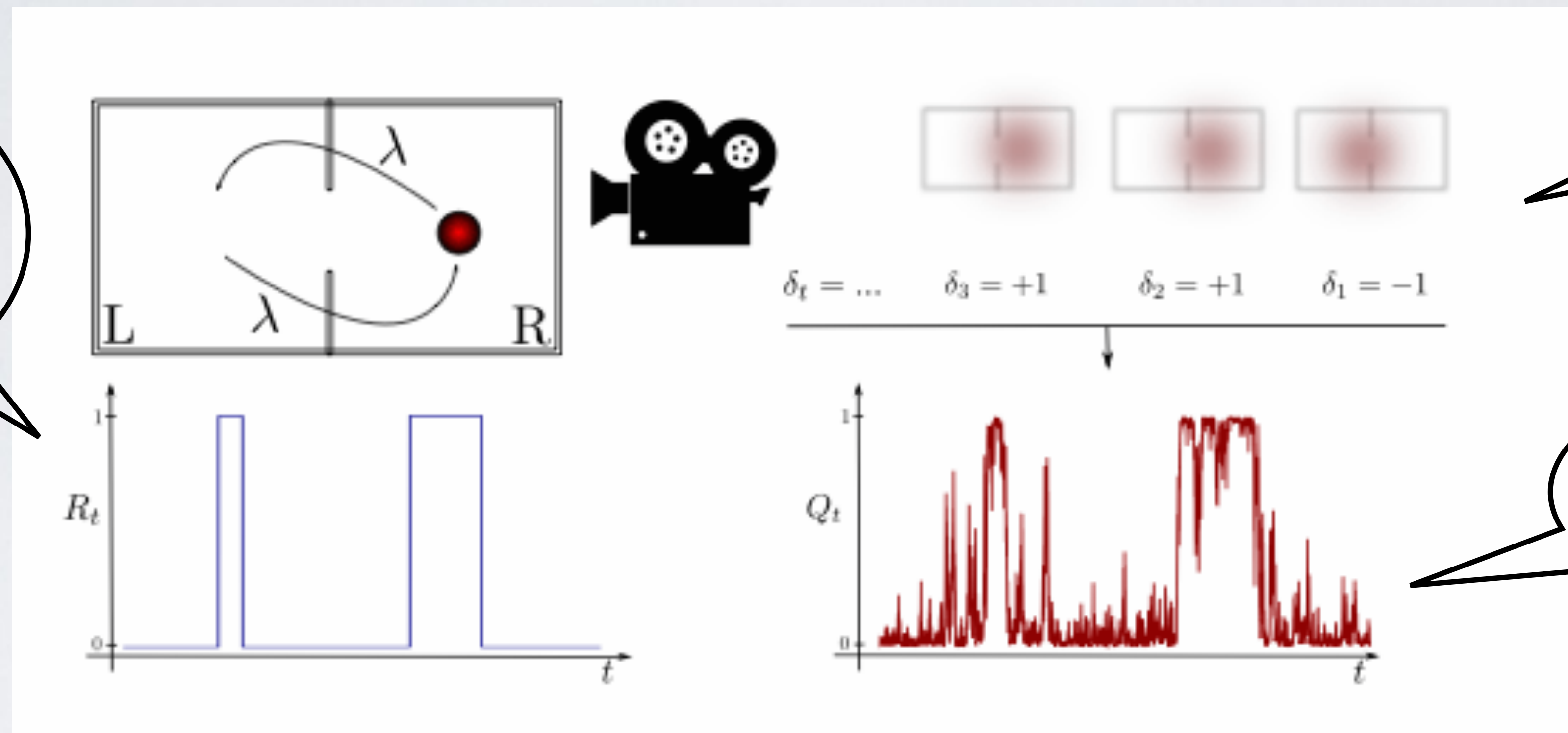
Amsterdam, June 2015

A Classical Toy Model: ‘Bayesian’ measurements

— Imagine a ‘classical’ particle in a box, with a probability to hope from left to right and back.
One ‘**observes**’ the system by taking blurry photos
and ‘**estimates**’ the particle position from the photos.

the real particle position

Left: R=1
Right: R=0



the photo’s data

the estimated position Q

— Bad photos => some probability to have ‘(un)-correct’ information on the particle position:

$$\mathbb{P}(\delta = 1|\text{particle on the left}) = \frac{1+\epsilon}{2}, \; \mathbb{P}(\delta = 1|\text{particle on the right}) = \frac{1-\epsilon}{2}$$

Epsilon codes how the value of delta is correlated to that of true position R.

— Estimated position (at time n given the past photo’s data):

$$Q_n := \mathbb{P}(\text{particle on the left at time } n \big| \text{pictures before } n)$$

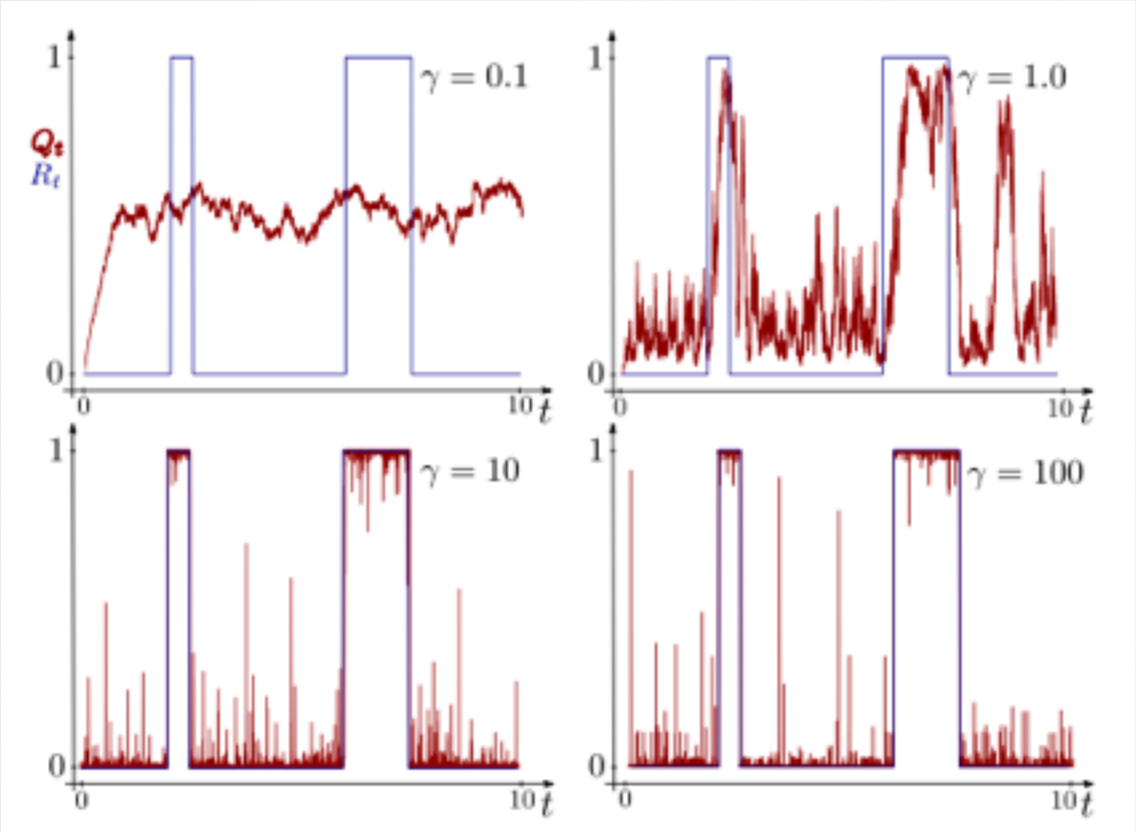
— The estimated positions are **recursively reconstructed** from the photo data and **Bayes rules**:

$$Q_{n+1} = \frac{\mathbb{P}(\delta_{n+1}|R_{n+1}=1)\mathbb{P}(R_{n+1}=1|\{\delta_k\}_{k\leq n})}{\mathbb{P}(\delta_{n+1}|\{\delta_k\}_{k\leq n})}$$

$$\mathbb{P}(R_{n+1}=1|\{\delta_k\}_{k\leq n}) = (1-\lambda)Q_n + \lambda(1-Q_n)$$

with lambda the probability to jump from right to left.

— Good information on the position for Q close to 0 or to 1.
Epsilon codes how information is acquired.

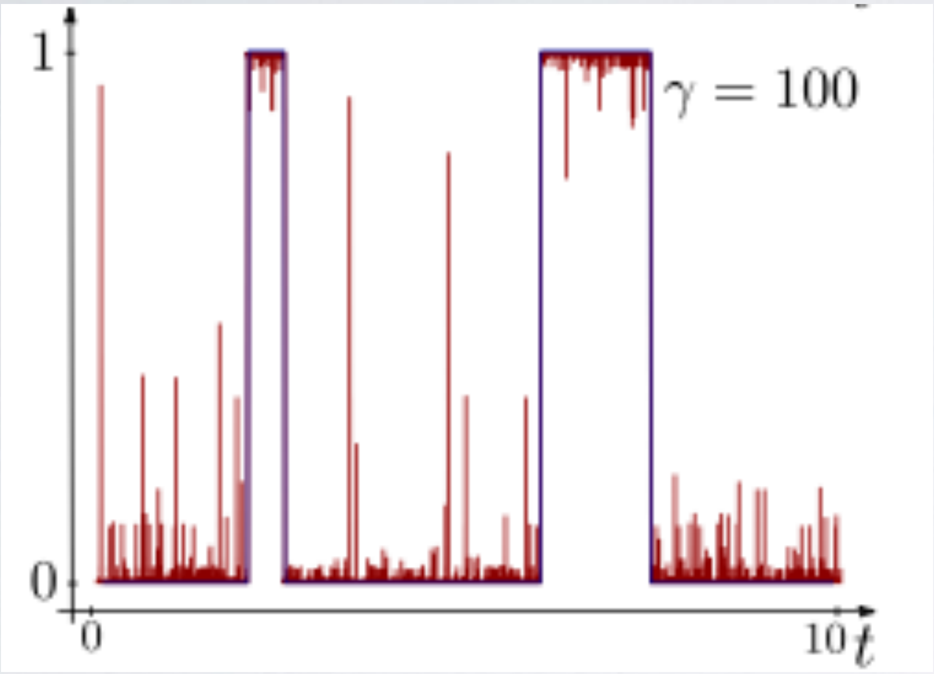


- At low **information rate**, no much information in Q.
- **Jumps** appear when the information rate increase.
- **Spikes** survive at extremely large information rates.

— Plots done in the scaling limit: $\epsilon \simeq \gamma\sqrt{\delta t},\ t \simeq n\,\delta t$ with $\delta t \to 0$
Notice: at each (discrete) step the amount of **extracted information is low**.

Spikes survive...

— **Spikes are fluctuations of the estimated value** 'Q' around the 'real' value 'R'.
They survive at infinite information rate: **Jumps are always dressed with spikes**.



They have a scale invariant statistics.
They form a **Point Poisson Process** with intensity (for spikes emerging from Q=0).

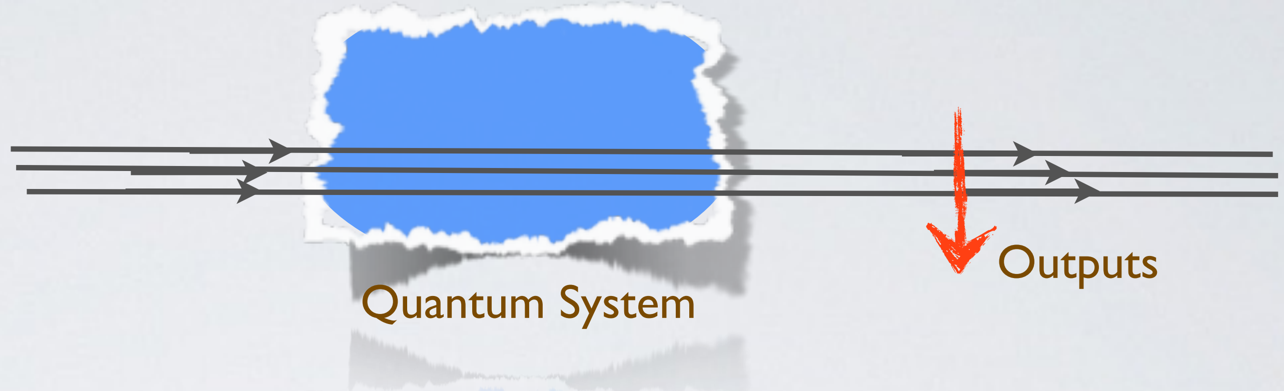
$$d\nu = \tilde{\lambda}\, \frac{dQ}{Q^2}\, dt$$

— In this classical model, there is a clear notion of what is the 'real' particle position (this is R).
The fluctuations are in the 'estimated' particle position.

— **What about for the quantum systems?**
-> First how to continuously monitor a quantum system (avoiding the Zeno effect)?
-> What is the statistics of the outputs?
I.e. How informations is extracted? What governs it? Does it show jumps and spikes?...

Monitoring quantum systems :

— **Monitoring:**
Time continuous indirect (weak) measurements.

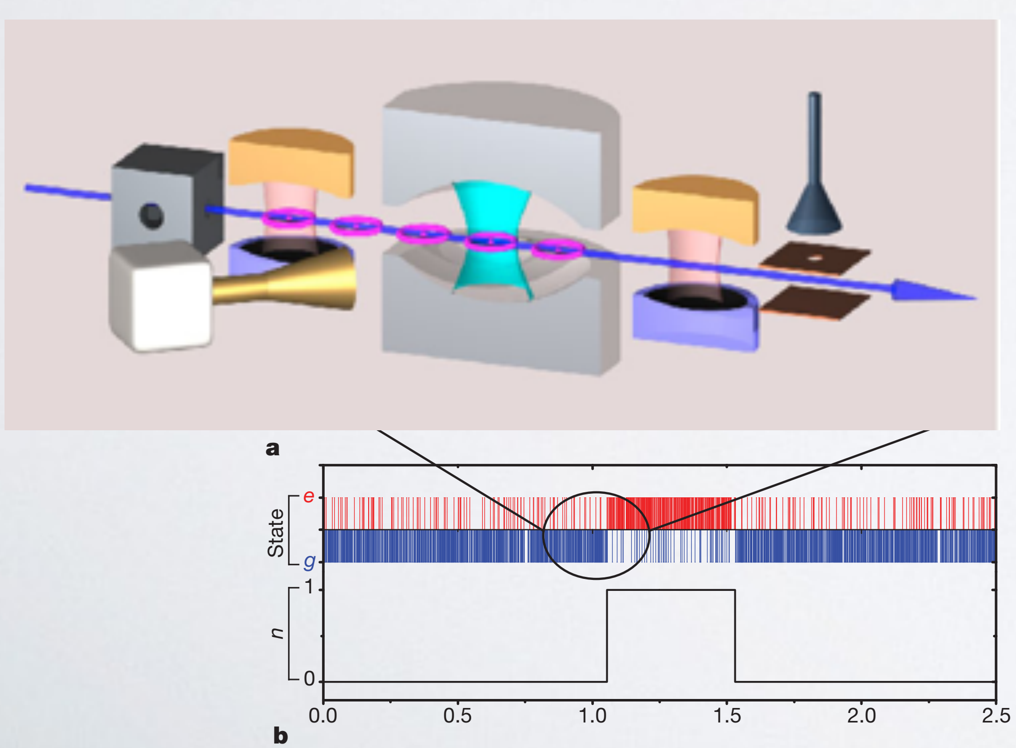


— Non-demolition (weak) measurements may be used **to observe a quantum system continuously in time** (and avoiding freezing by the **quantum Zeno effect**).
They are keys to manipulate and **control quantum systems**.

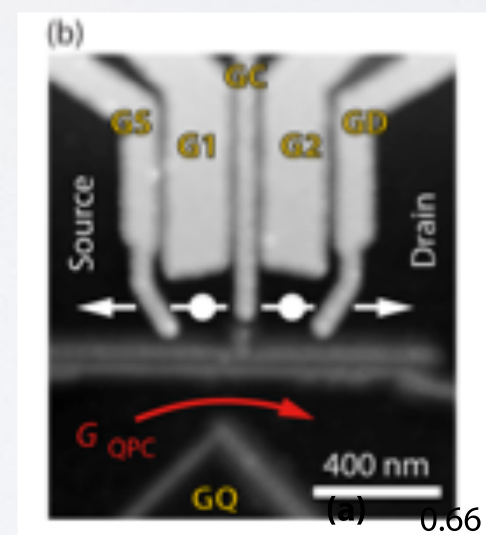
— In cavity QED :

Quantum jumps of light recording the birth and death of a photon in a cavity Nature 446, 297 (2007).

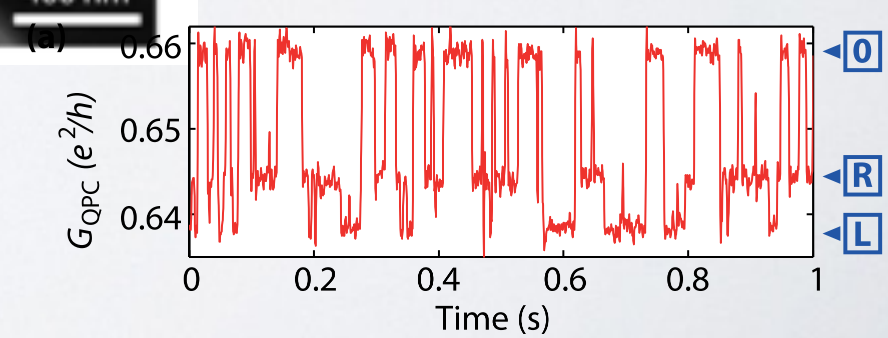
Sébastien Gleyzes[1], Stefan Kuhr[1]†, Christine Guerlin[1], Julien Bernu[1], Samuel Deléglise[1], Ulrich Busk Hoff[1], Michel Brune[1], Jean-Michel Raimond[1] & Serge Haroche[1,2]



— In mesoscopic quantum systems, e.g. quantum dots or circuit QED:



J. Appl. Phys 113, 136507 (2013)



Quantum Jumps…

— **Know from Bohr's original atomic model** (then - 1913)

« **Abrupt transitions** » between stable orbitals
with emission of light or energy quanta.

***On the Constitution of Atoms and Molecules*, N. Bohr, *Philos. Mag.* 26, 1 (1913).**

(1) That the dynamical equilibrium of the systems in the stationary states can be discussed by help of the ordinary mechanics, while the passing of the systems between different stationary states cannot be treated on that basis.

(2) That the latter is followed by the emission of a *homogeneous* radiation, for which the relation between the frequency and the amount of energy emitted is the one given by Planck's theory.

— **First observed** in atomic fluorescence in **1986**,

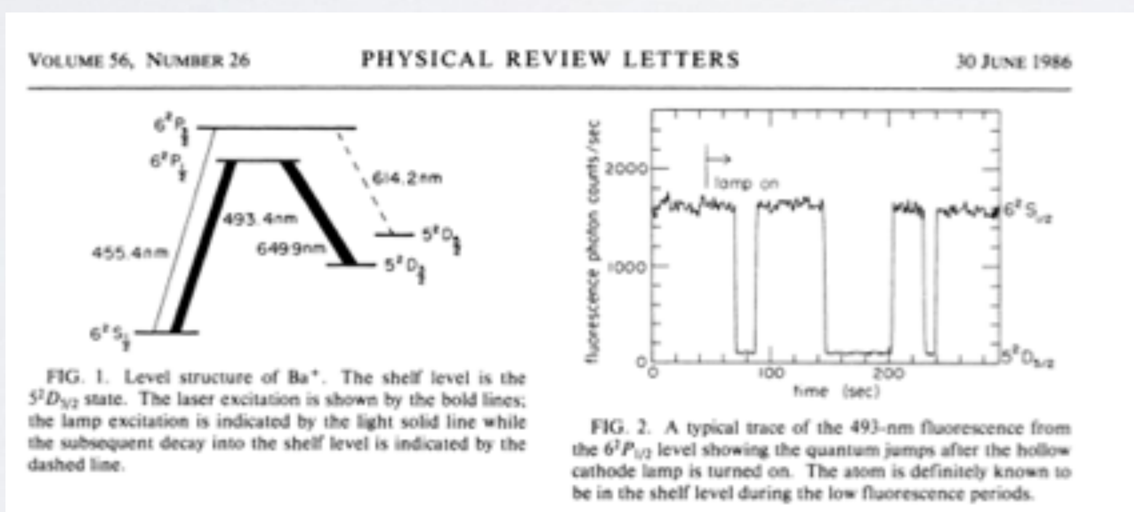

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FIG. 1. Level structure of Ba^+. The shelf level is the $5^2D_{5/2}$ state. The laser excitation is shown by the bold lines; the lamp excitation is indicated by the light solid line while the subsequent decay into the shelf level is indicated by the dashed line.

FIG. 2. A typical trace of the 493-nm fluorescence from the $6^2P_{1/2}$ level showing the quantum jumps after the hollow cathode lamp is turned on. The atom is definitely known to be in the shelf level during the low fluorescence periods.

W. Nagourney et al, PRL 56, 2797 (1986).
Th. Sauter et al, PRL 57, 1696 (1986)
J,C. Bergquist et al, PRL 57, 1699 (1986).

— **A bit more….** **Quantum spikes and Applications….**

How to model continuous quantum monitoring?

— Could be modeled by **alternative iteration of system evolution** (e.g. quantum dynamical map) **and discrete repeated weak measurements** (with short time interval).
It can be done in the discrete setting. The time continuous formulation is simpler to analyse.
=> « time continuous measurement » in Q.M. Belavkin, Barchielli, Milburn-Wiseman,…..

— This yields a **competition between the deterministic system evolution and the random evolution due to weak quantum measurement**.

$$d\rho = (d\rho)_{sys} + (d\rho)_{meas}, \quad \text{during time } dt$$

The **first** term **is** a **deterministic**, unitary or dissipative, system evolution.
The **second is** the **random** evolution due to the measurement back-action.

$$\boxed{\begin{aligned}(d\rho)_{\rm sys} &= \big(- i[H_{\rm sys},\rho] + L_{\rm dissp}(\rho)\big)\, dt\\ (d\rho)_{\rm meas} &= \sigma^2\, L_{\rm meas}(\rho)\, dt + \sigma\, D_{\rm meas}(\rho)\, dB_t\end{aligned}}$$

A **Brownian motion**, coding for all probe measurements.

with
$$\left|\begin{array}{l} L_N(\rho) := N\rho N^\dagger - \frac{1}{2}(N^\dagger N\rho + \rho N^\dagger N)\\ D_N(\rho) := N\rho + \rho N^\dagger - \rho\, U_N(\rho) \text{ and } U_N(\rho_t) := \mathrm{tr}(N\rho + \rho N^\dagger)\end{array}\right.$$

They depend on which observable is monitored (here O=N+N*).
Sigma codes for the strength of the indirect measurements (**measurement time scale**).

Example: A (coherent) qubit.

— A two-state system, (Q-bit):
Monitoring an observable **not commuting with the hamiltonian** of a two-state system.
There a **two time scales**: that associated to the unitary evolution and that to the measurement.
The **two processes** are in competition:

$$d\rho_t = - i\frac{\Omega}{2}[\sigma_y, \rho_t]dt - \frac{\gamma^2}{2}\left[\sigma_z\left[\sigma_z, \rho_t\right]\right]dt \\ + \gamma\left(\sigma_z\rho_t + \rho_t\sigma_z - 2\mathrm{tr}[\sigma_z\rho_t]\right)dW_t,$$

Hamiltonian: $H = \frac{\Omega}{2}\sigma_y$

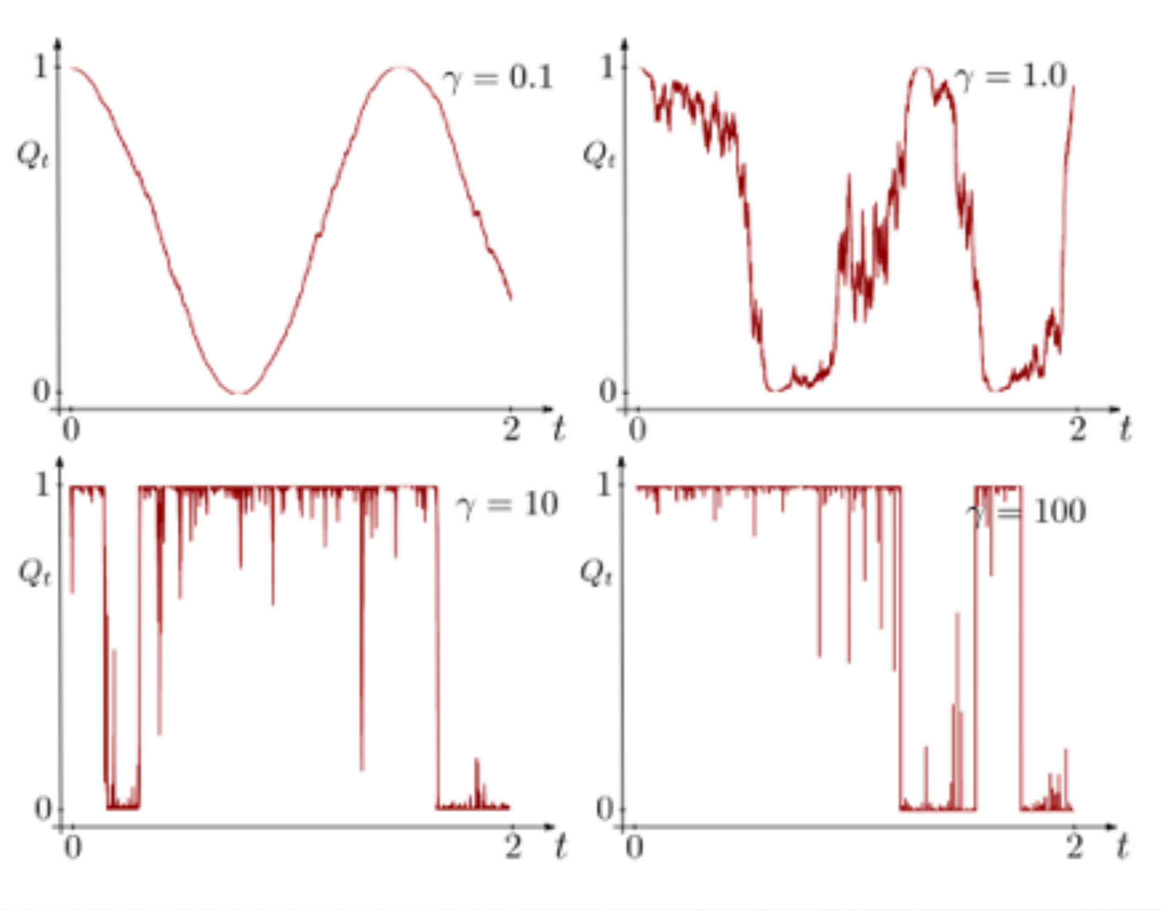
Observable: $\mathcal{O} = \sigma_z$

— Evolution of Q with increasing information rate (gamma)

The system state stays « pure ».
Let $Q_t := \langle +|_z\, \rho_t\, |+\rangle_z$
be the 'population'.

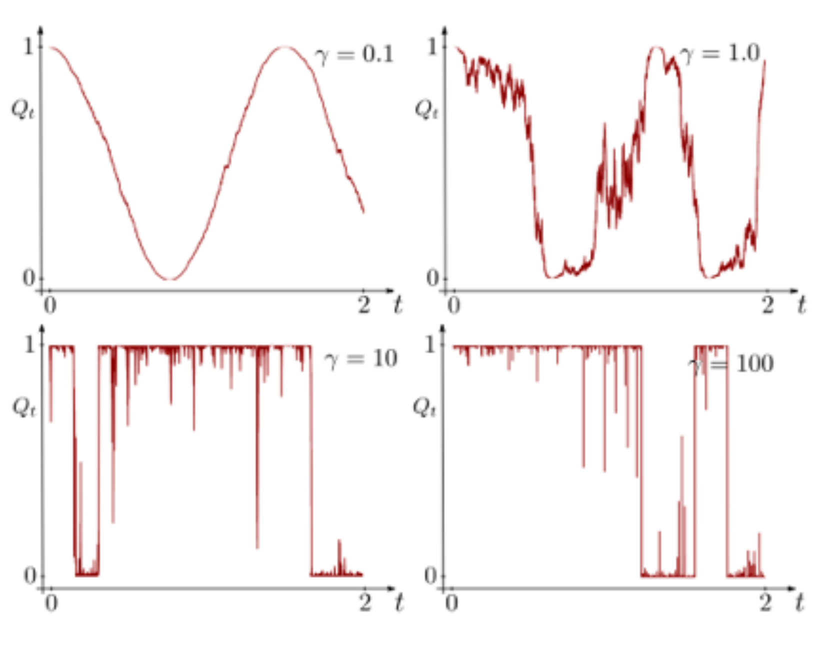
Two time scales:

$$\tau_{\mathrm{meas}} := \gamma^{-2}$$
$$\tau_{\mathrm{evol}} := \Omega^{-1}$$



Spikes survive...

••• the limit of infinite information rate (gamma infinite).



— The mean time in between jump is:
(Zeno freezing) $\tau_{\text{flip}} = \tau_{\text{evol}}^2/\tau_{\text{meas}} = (\gamma/\Omega)^2$

— **Claim**: In the large information rate limit, the spikes form a **Point Poisson Process** with intensity: $d\nu = \omega^2 \frac{dQ}{Q^2}\, dt$
(for spikes emerging from Q=0, with $\Omega = \gamma\,\omega$)

— Spike fluctuations in the monitoring of a coherent qu-bit are **identical** to those present in the classical model (only the time scale changes)! Even-though the state is always pure and there is no obvious 'reality' variable as R,...

— Actually, the equations for the classical model are also those for the energy monitoring of qu-bit in contact with a thermal bath.

$$dQ_t = \tilde\lambda(p - Q_t)\, dt + \gamma Q_t(1 - Q_t)\, dW_t$$

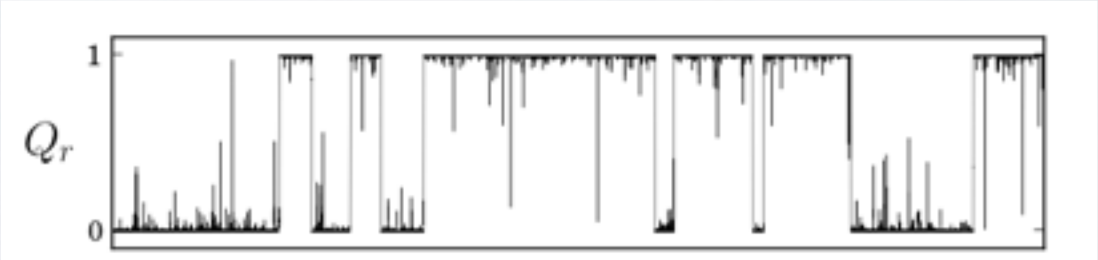
⟶ **Universality in the spikes statistics (?)......**

What is the strong limit of weak measurement?

— The jumps and spike statistics is encoded into the stochastic differential equations of the density matrix quantum trajectories

$$d\rho = (d\rho)_{sys} + (d\rho)_{meas},$$ - with $$\begin{aligned}(d\rho)_{sys} &= \big(-i[H,\rho] + L_{dissip}(\rho)\big)dt \\ (d\rho)_{meas} &= \sigma^2\, L_{meas}(\rho)dt + \sigma\, D_{meas}(\rho)dW_t\end{aligned}$$

The strong measurement limit is a strong noise limit (in contrast with Kramer's theory).



— Strong measurement collapses the system on the pointer states. Other dynamical processes induce **jumps from one pointer state to the other**. They form a Markov process.

— **Claim/Conjecture**:

— **At strong coupling**, these processes converge (weakly) to finite state **Markov chains**:
All N-point functions converge to that of a specified Markov chain on the pointer states.

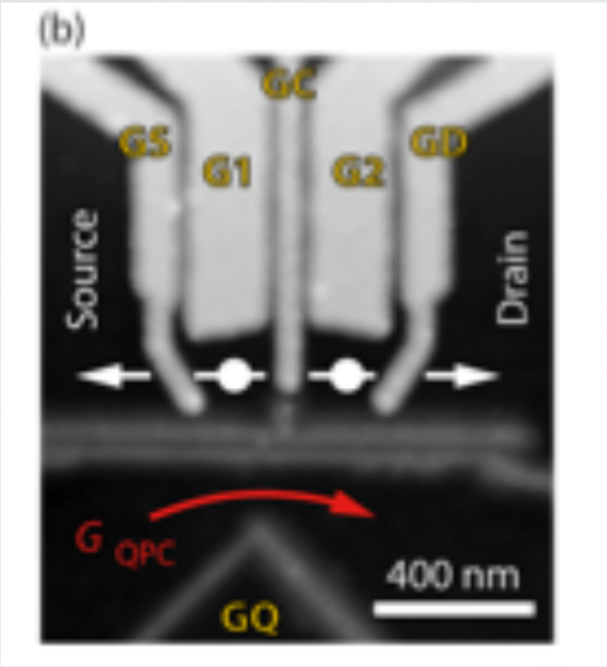
— **The spikes survive** in the strong coupling limit (hence the weakness of the limit).
Their statistics are encoded into **Poisson point processes** in [0,1]xR.

— The spikes statistics is **universal**, in the sense that they are independent of the dynamical process which generate them and apply to any (finite dimensional) systems.

Application: a mesoscopic Maxwell Daemon…
… or control through measurement.

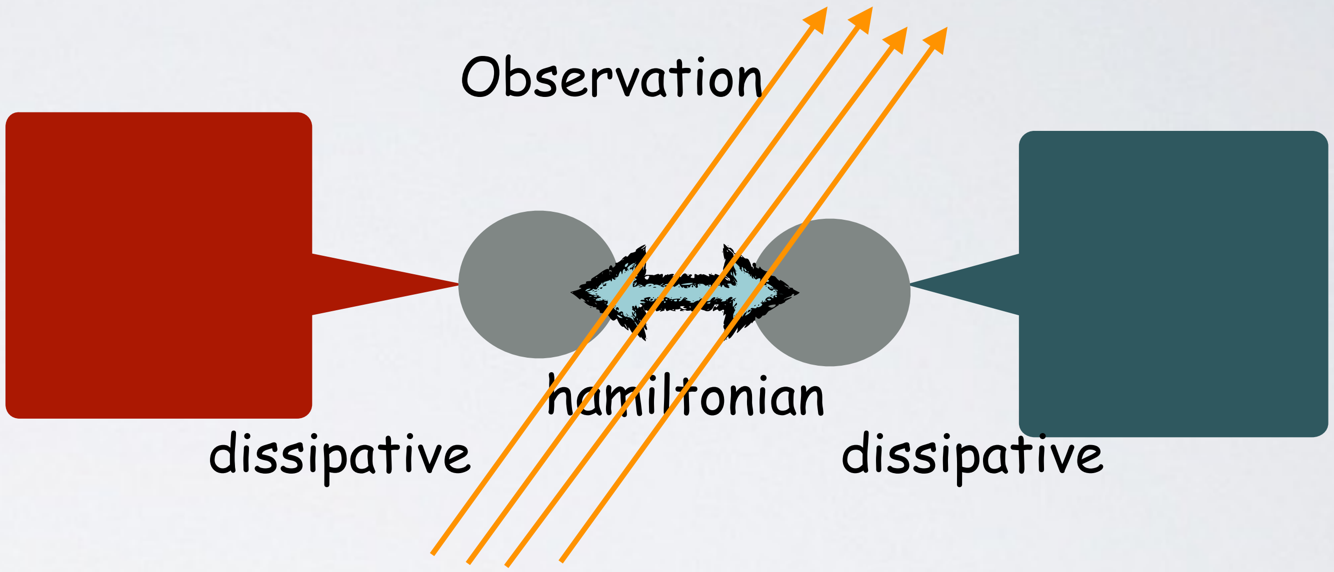
— **Double quantum dot** (DQD).

Baths
+
System



DQD measurement device,
via QPC conductivity.

— **Its (abstract) idealisation**:
(but other configuration possible, e.g cQED)

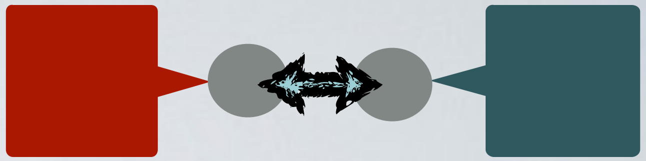


— To control we need **to observe** (continuously), **to get information** (continuously), **to back-act** on the system (continuously)

— **By controlling**: generate a flux, even for reservoirs at equal chemical potential, by adapting the measurement strength to the information we get.

— **The principle**: **Back-action** of the measurement is very **different** in system evolving **unitarily or dissipatively**.

Controlling through measurement:



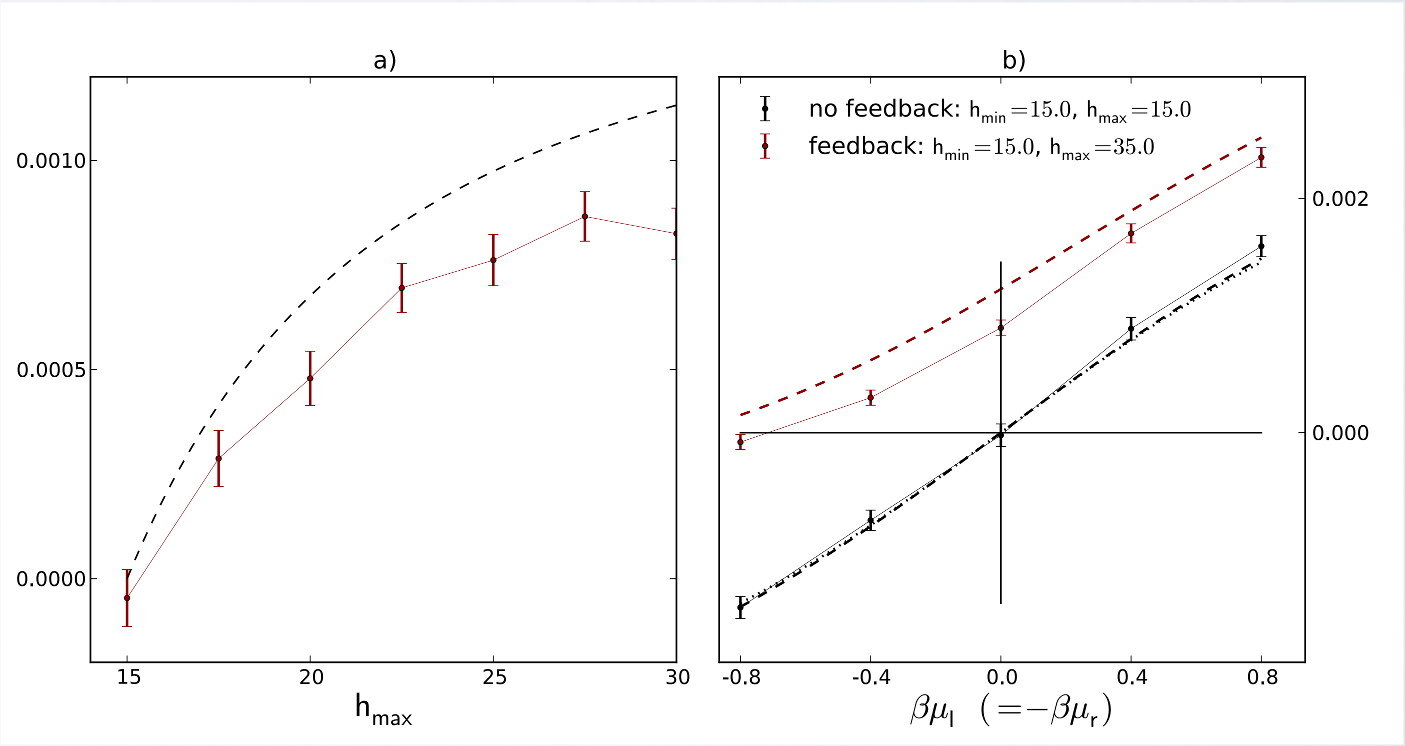
— How to **control quantum fluxes?… through the measurement back-action**:
Hamiltonian and **dissipative dynamics** « **react** » very **differently** under measurement:
one is « Zeno frozen », the other not.
This allows **to select (open/close) possible dynamical pathways/processes,**
hence a kind of a mesoscopic Maxwell daemon…

— By **changing the intensity of the measurement depending on the information** on the electron position one has, one may control the electron flux.
For instance, we may measure more strongly when it is « known » that the electron is on the right dot and lightly when it is « believed » to be on the left.

$$\langle J\rangle_{stat} \simeq \frac{u^2}{(m_l+m_r+1)}\left(\frac{m_l}{h^2_{min}} - \frac{m_r}{h^2_{max}}\right)$$

with h(min/max) the measurement strength
and log(m(l/r)) the chemical potential.

This generates a net flux
from the left to the right,
even if the two reservoirs are symmetrically equally filled!!

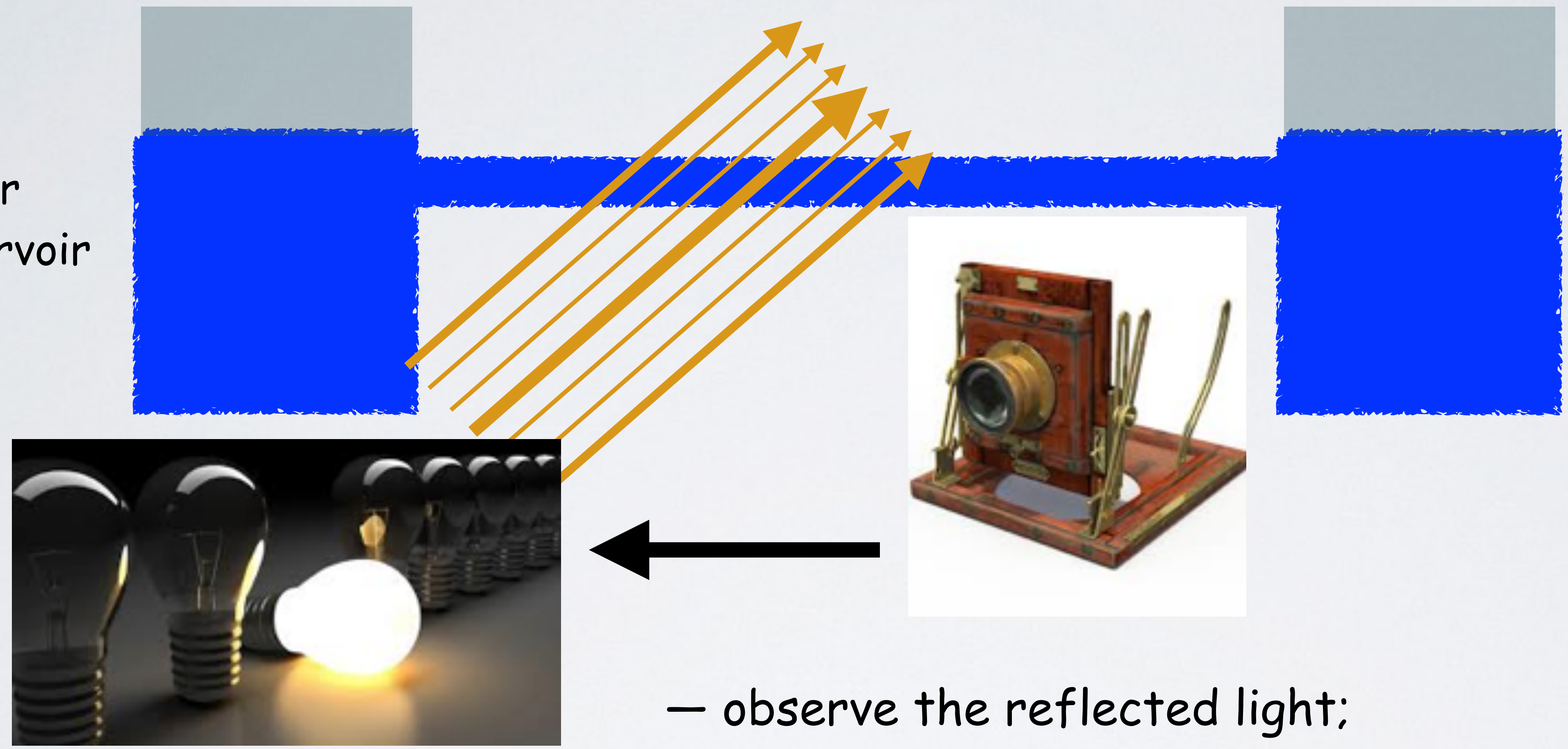


flux at equal chemical potential

flux as function of the chemical potential

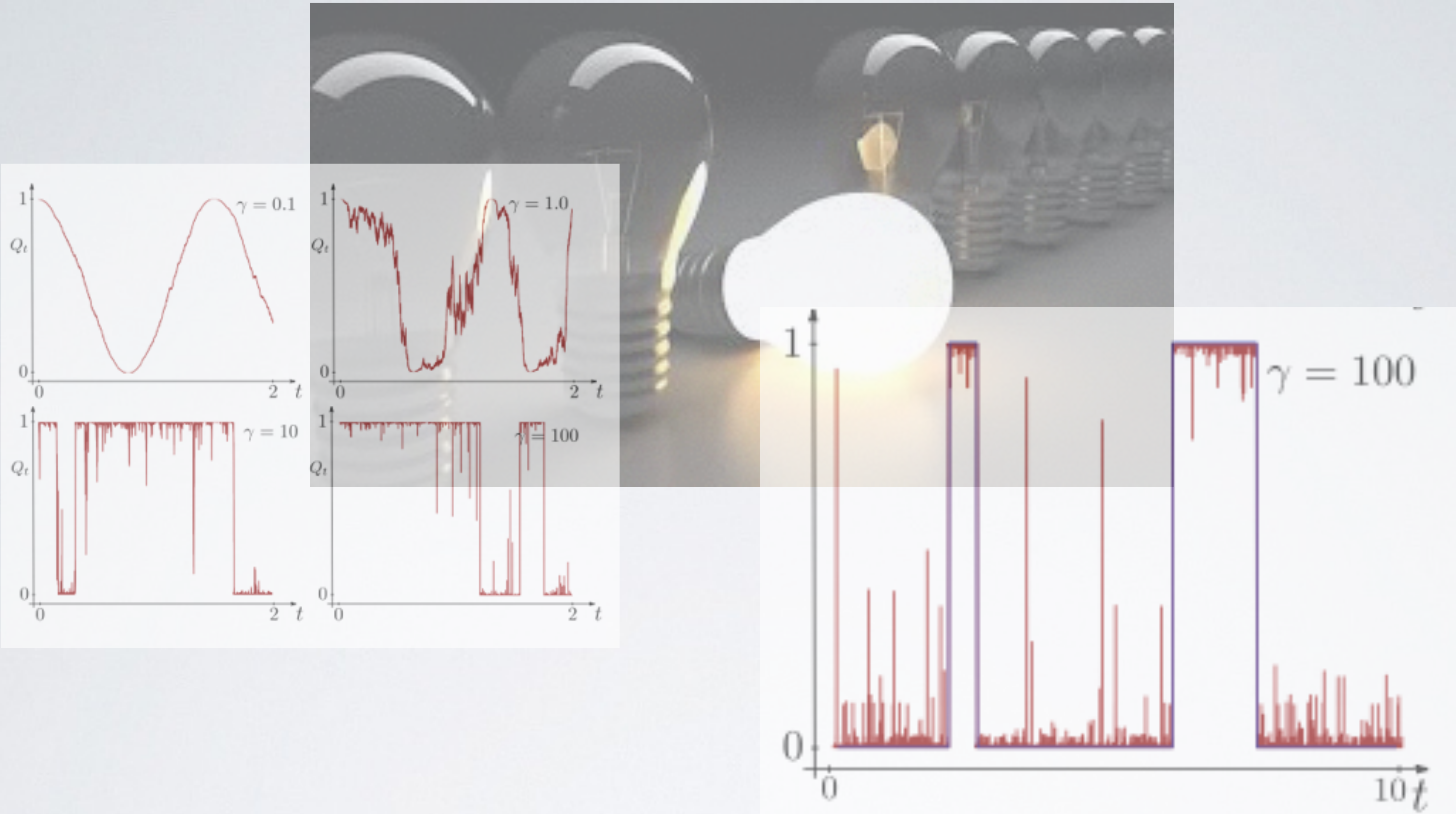
If a classical analogy is permitted...

— Two equally filled (water) reservoirs connected by a channel,
enlighten with adaptable light beams.



— observe the reflected light;
— adapt the intensity of the light accordingly;
— And, if the (classical) world was quantum mechanical…
one may then have the possibility
to generate a flux in between the reservoirs….

— Is the « Double Quantum Dots », or any other system, experimentally realizable?



Thank you.

How to code the jumps and spikes statistics? (II)

— The jumps and spike statistics can be computed from the SDE of the quantum trajectories, and these are not universal

$$d\rho_t = \big(i[H,\rho_t] + L_N(\rho_t)\big)dt + D_N(\rho_t)\,dB_t,$$

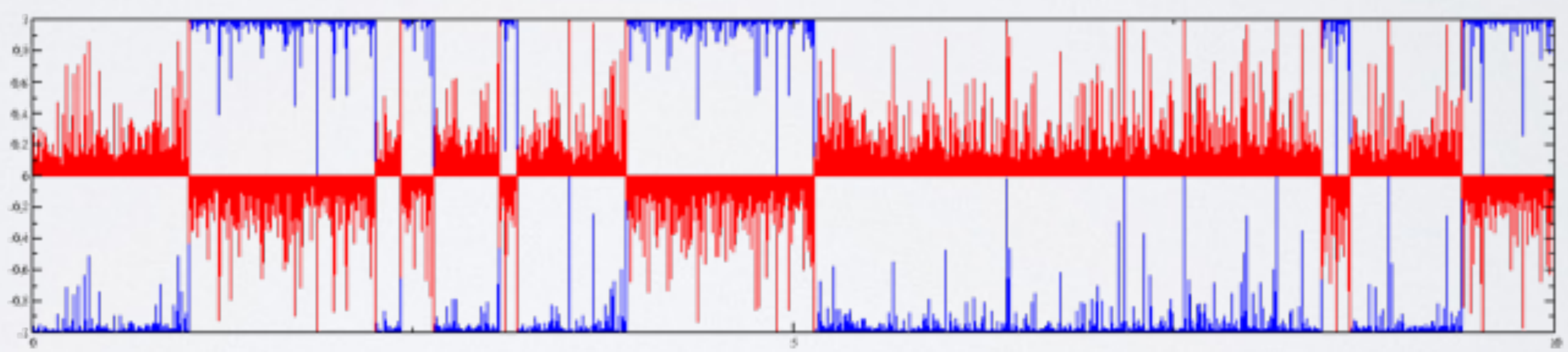
— But, asymptotically, the spike statistics is « **geometrical and universal** »:

> **Claim**: Let Q be the diagonal component of the density matrix.
> The **maximum (minima)** of Q on a **quantum trajectory** (at strong measurement) **form a point Poisson process with intensity**

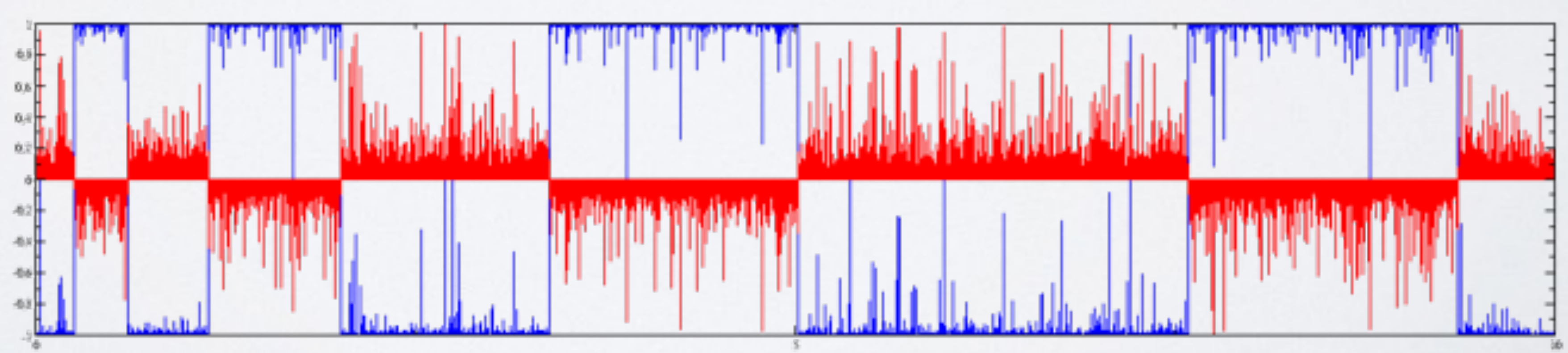
$$\boxed{d\nu = \lambda\, dt\, [\delta(1-Q)\, dQ + \tfrac{dQ}{Q^2}]}$$

— For spikes emerging from Q=0.

Reconstructed by solving the SDE



Reconstructed by using the Poisson point process



Details/Hints on the proof:

— Let D be the 2nd order differential operator associated to the quantum trajectory SDEs. After proper rescaling (avoiding the Q-Zeno effect), it decomposes as:

$\mathfrak{D} = \mathfrak{D}_0 + \sigma^2 \mathfrak{D}_2$ with D_2 the diff. op. associated to the measurement process.

At large measurement strength the process is projected on the kernel of D_2, the set of martingales for the measurement process, in one-to-one correspondance with the pointer states…. and then perturbation theory.

— The spike maxima/minima form a Poisson point process at large measurement strength. Explicit maxima/minima properties can be « directly » computed from the SDEs, say: $dQ_t = \lambda\,(p - Q_t)\,dt + \sigma\,Q_t(1-Q_t)\,dB_t$ at large sigma.

(i) The mean time duration between two successive jumps from 0 to 1, defined as the first stopping time to reach Q=1 starting from Q=0, is $1/\lambda(1-p)$; and symmetrically from 1 to 0.

(ii) For 0<x<Q<1, the probability for a trajectory starting x to reach Q before going back to 0 is x/Q.

The first statement fixes the typical time scale.
The second is about the distribution of the maximum height of the spikes starting from 0 and conditioned to be bigger than x. (Similarly for the spikes emerging from 1).

— These two properties fully determine the intensity of the Poisson process, because: $\mathbb{P}\Big[\mathcal{N}_{[x,Q]\times[0,\delta t]} = 0, \mathcal{N}_{[Q,1]\times[0,\delta t]} = 1 \Big| \mathcal{N}_{[x,1]\times[0,\delta t]} = 1\Big] = \frac{\nu_0([Q,1]\times[0,\delta t])}{\nu_0([x,1]\times[0,\delta t])} = \frac{x}{Q}.$

— Similar arguments apply in the case of Rabi oscillations/Q-jumps.

— Argue that the general case can be reduced to this case (proof only in the « diagonal » case).