Topological phenomena in periodically driven systems: disorder and interactions

Erez Berg

Mark Rudner (Copenhagen), Netanel Lindner (Technion), Paraj Titum and Gil Refael (Caltech), Michael Levin (Chicago), Takuya Kitagawa and Eugene Demler (Harvard)

מכון ויצמן למדע

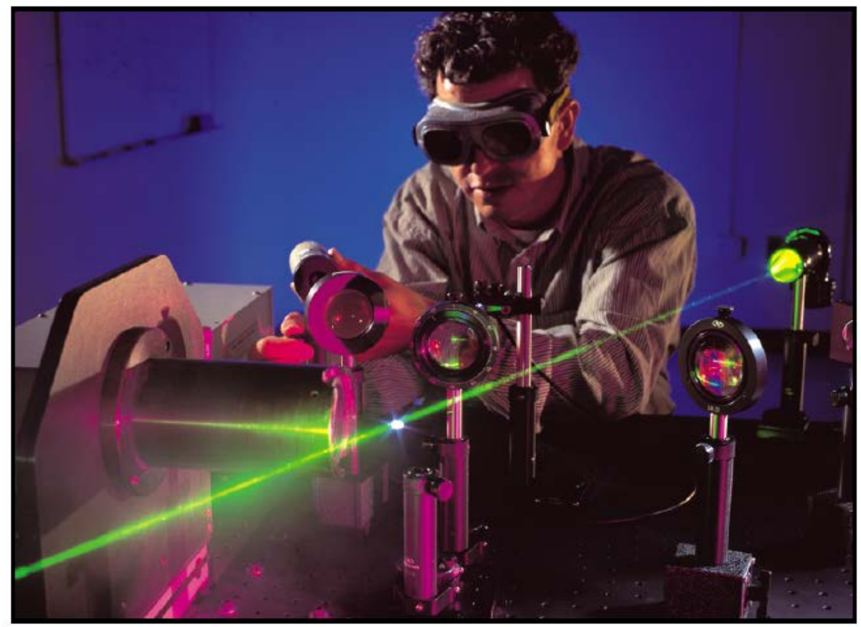
Weizmann Institute of Science

New types of non-equilibrium phenomena in periodically-driven systems?

~1200

$Pb \rightarrow Au$

2015



$Si \rightarrow HgTe?$

$\rightarrow \; \ldots \; ?$

Outline

- **Introduction**: Floquet, Bloch, Floquet-Bloch
- **Disorder** and the Anomalous Floquet-Anderson Insulator (AFAI)
- **Interactions** and quasi-steady states

Floquet states and the quasi-energy

$$i\frac{d}{dt}|\psi\rangle = H(t)|\psi\rangle; \quad H(t+T) = H(t)$$

"Stroboscopic" evolution operator:

$$U(T) = \mathcal{T} e^{-i\int_0^T H(t)dt}$$

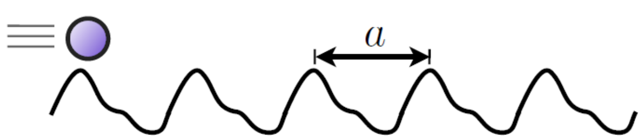
Diagonalize:

$$U(T)|\psi_n\rangle = e^{-i\varepsilon_n T}|\psi_n\rangle$$

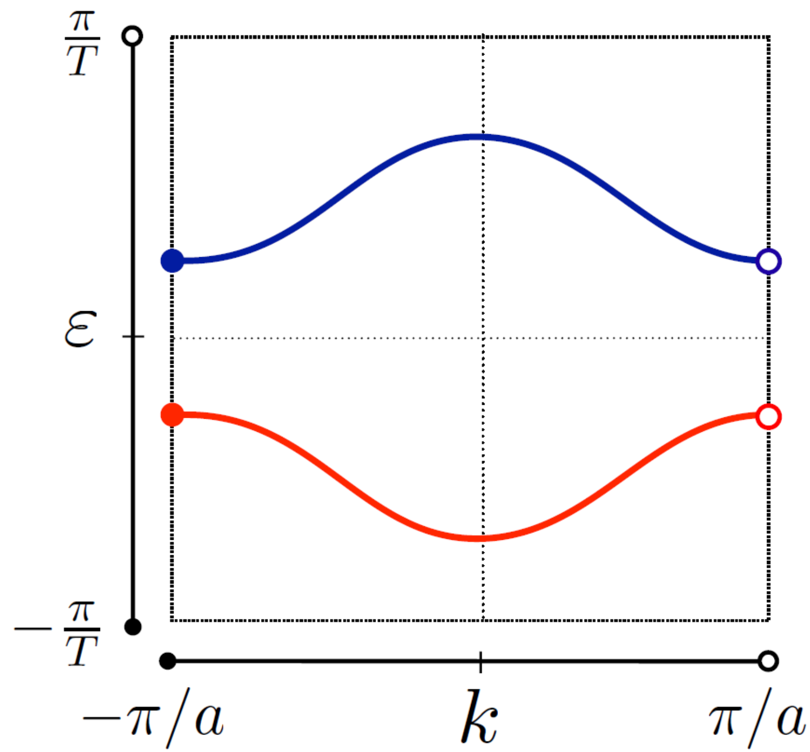
Eigenvalue invariant under $\varepsilon_n \to \varepsilon_n + 2\pi N/T$: quasi-energy lives on a circle

Floquet-Bloch Bands

- On a lattice: Floquet-Bloch bands, similar to static systems:



$$V(x+a) = V(x)$$



- Suggests topological phenomena analogous to those found in static systems.

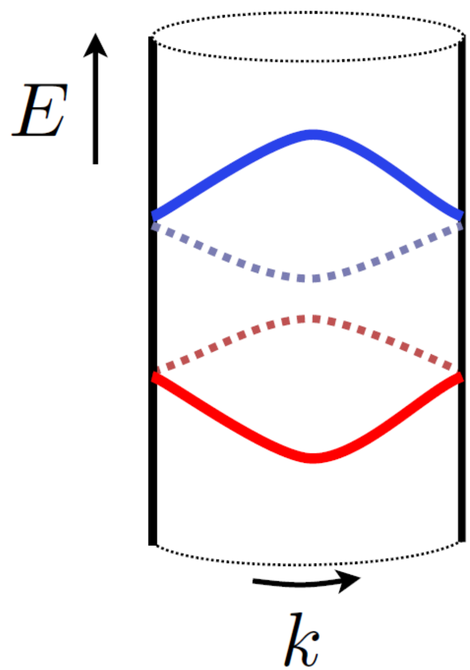
T. Kitagawa, EB, M. Rudner, E. Demler PRB (2010)

Theoretical proposals:
Aoki, Oka, Lindner, Galitsky, Refael, Gu, Arovas, Fertig, Auerbach, Kitagawa, Brataas, Fu, Demler, Kundu, Seradjeh,…

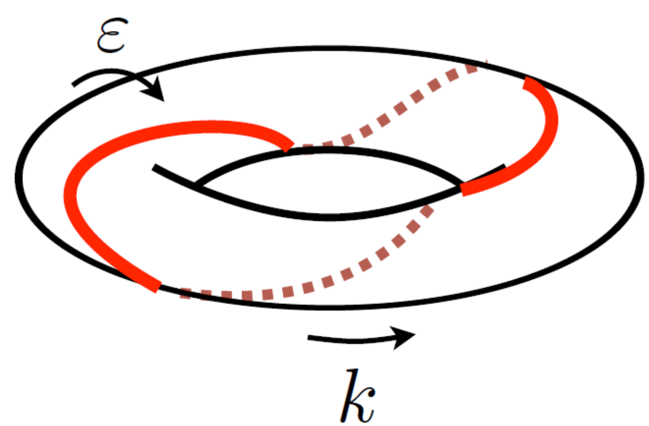
Experiments:
Wang et al. (2013): Irradiated Bi2Se3 surface
Jotzu et al. (2014): Cold atoms: Floquet realization of the Haldane model

New topological configurations possible in driven systems

Normal band structure: <u>cylinder</u>



Quasi-band structure: <u>torus</u>



Quasi-energy winding related to quantized adiabatic transport

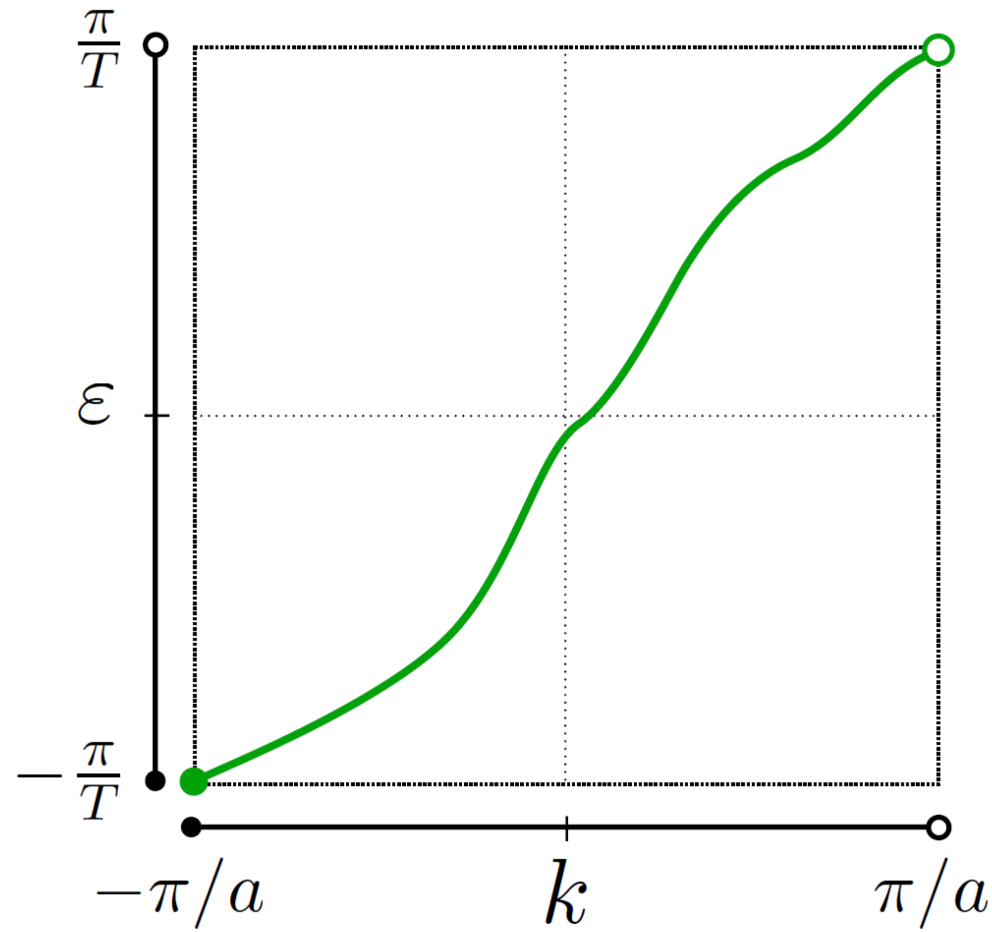
Average group velocity quantized

$$\bar{v}_g = \overline{\frac{d\varepsilon_k}{dk}} = a/T$$

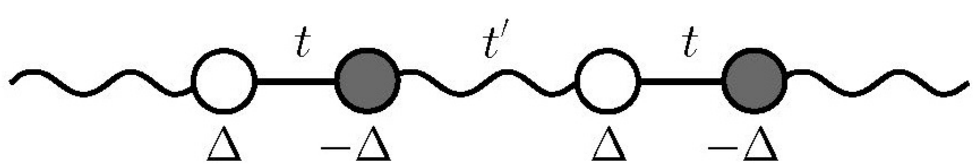
Average displacement:

$$\overline{\Delta x} = \bar{v}_g T = a$$

shift by one unit cell



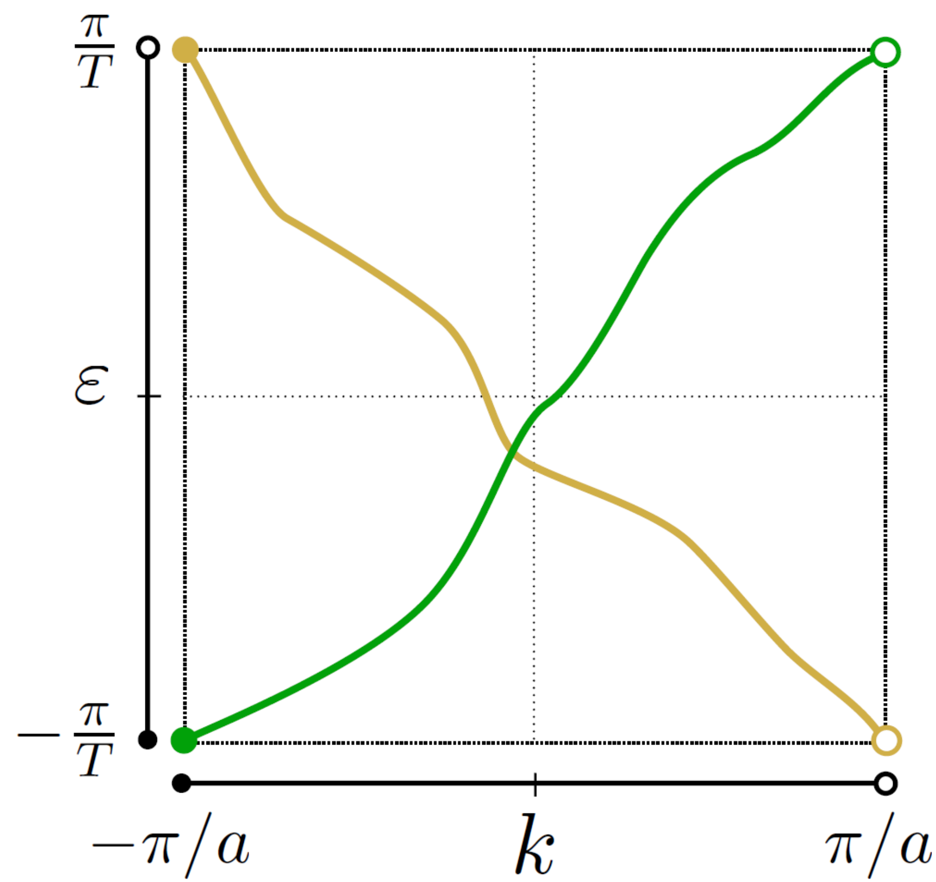
Adiabatic pump
(Thouless 83')



Counter-propagating modes decouple in adiabatic limit

Winding number

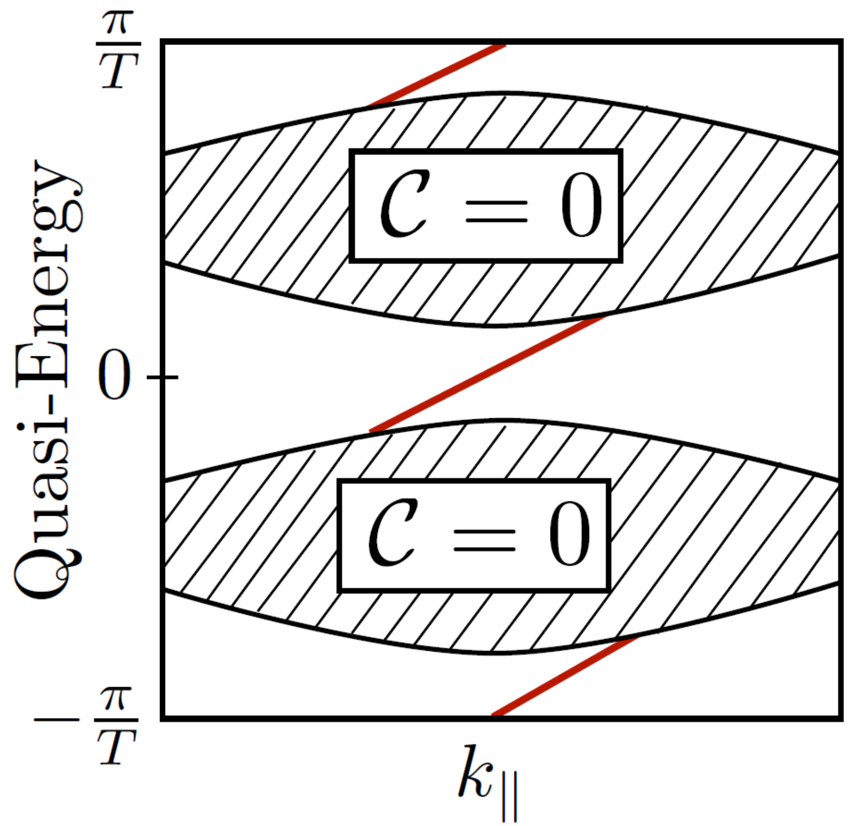
$$W = \oint \frac{dk}{2\pi} \, \mathrm{Tr} \left[U^{-1} \partial_k U \right]$$

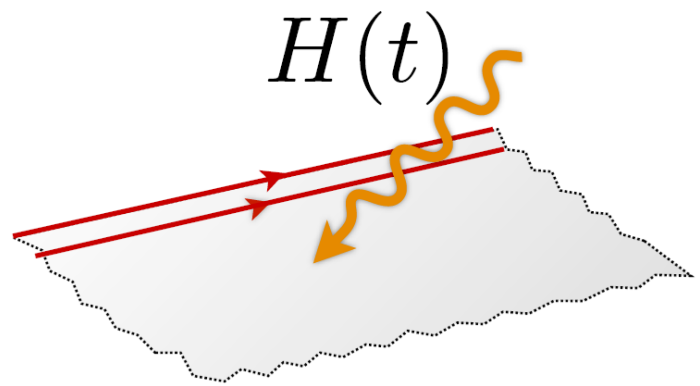


Note: net winding number of all bands must be zero

Two Dimensions

Driven 2D systems may support chiral edge modes even when all Chern numbers are <u>zero</u>!

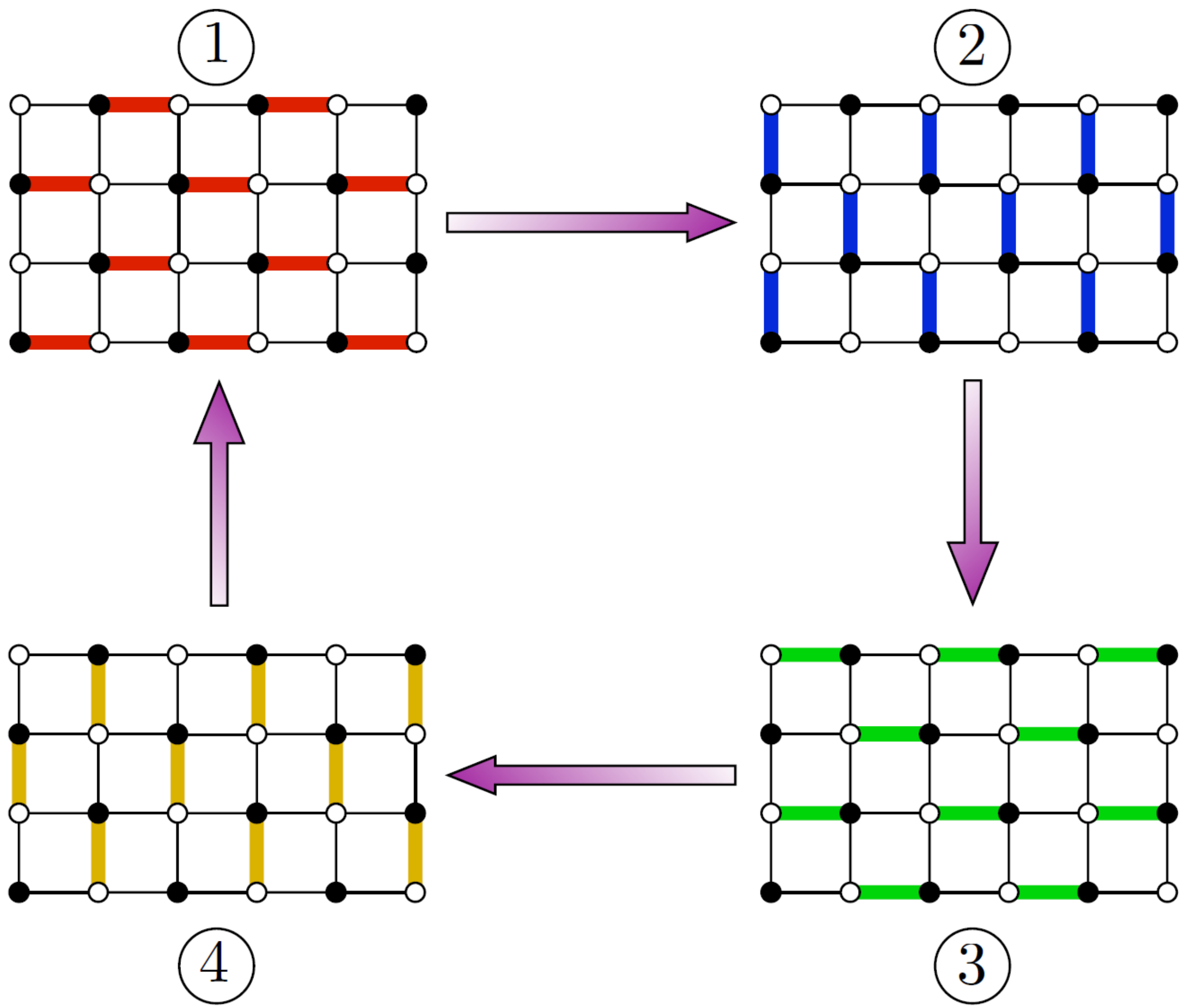




Chiral edge modes for $\mathcal{C} = 0$ bands

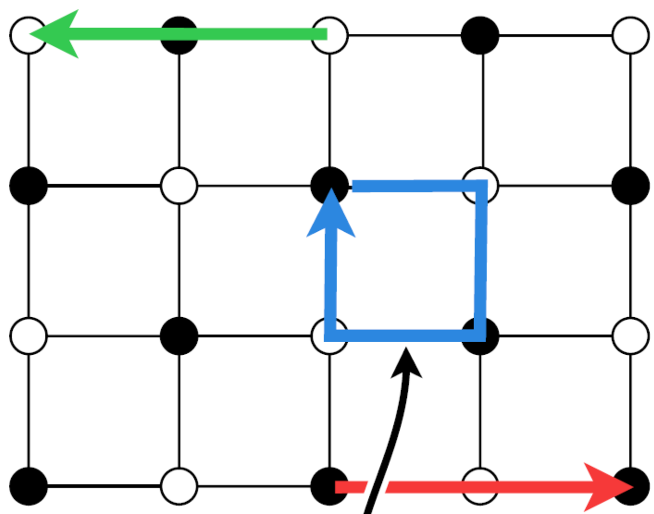
T. Kitagawa, EB, M. Rudner, and E. Demler PRB (2010)
M. Rudner, N. Lindner, EB, and M. Levin, PRX (2013)

New phase illustrated by model with modulated hoppings

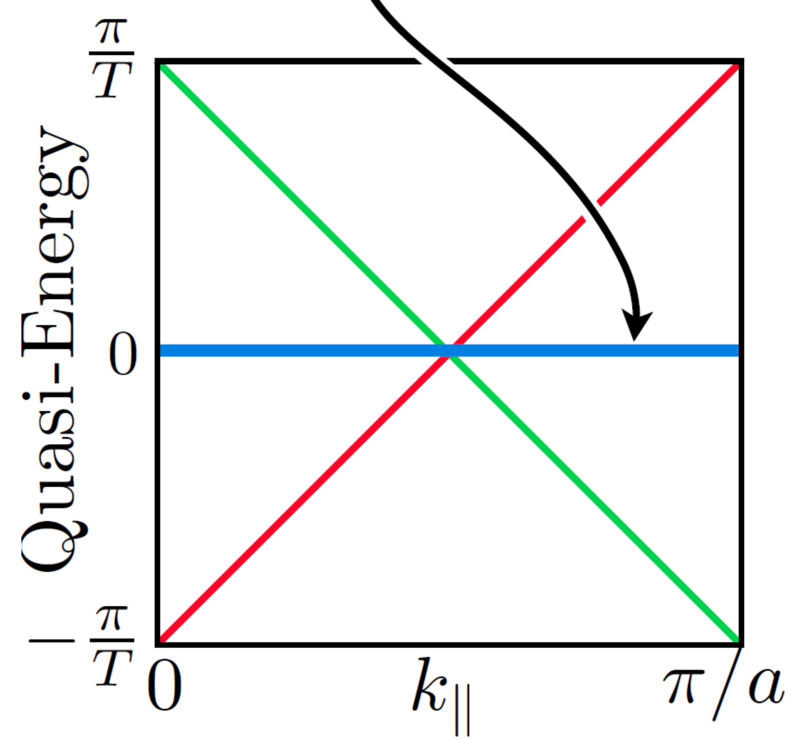


Bulk evolution trivial, chiral modes propagate along edges

Bulk Floquet operator is trivial: $U(T) = \mathbb{1}$



Particle returns to initial position after cycle

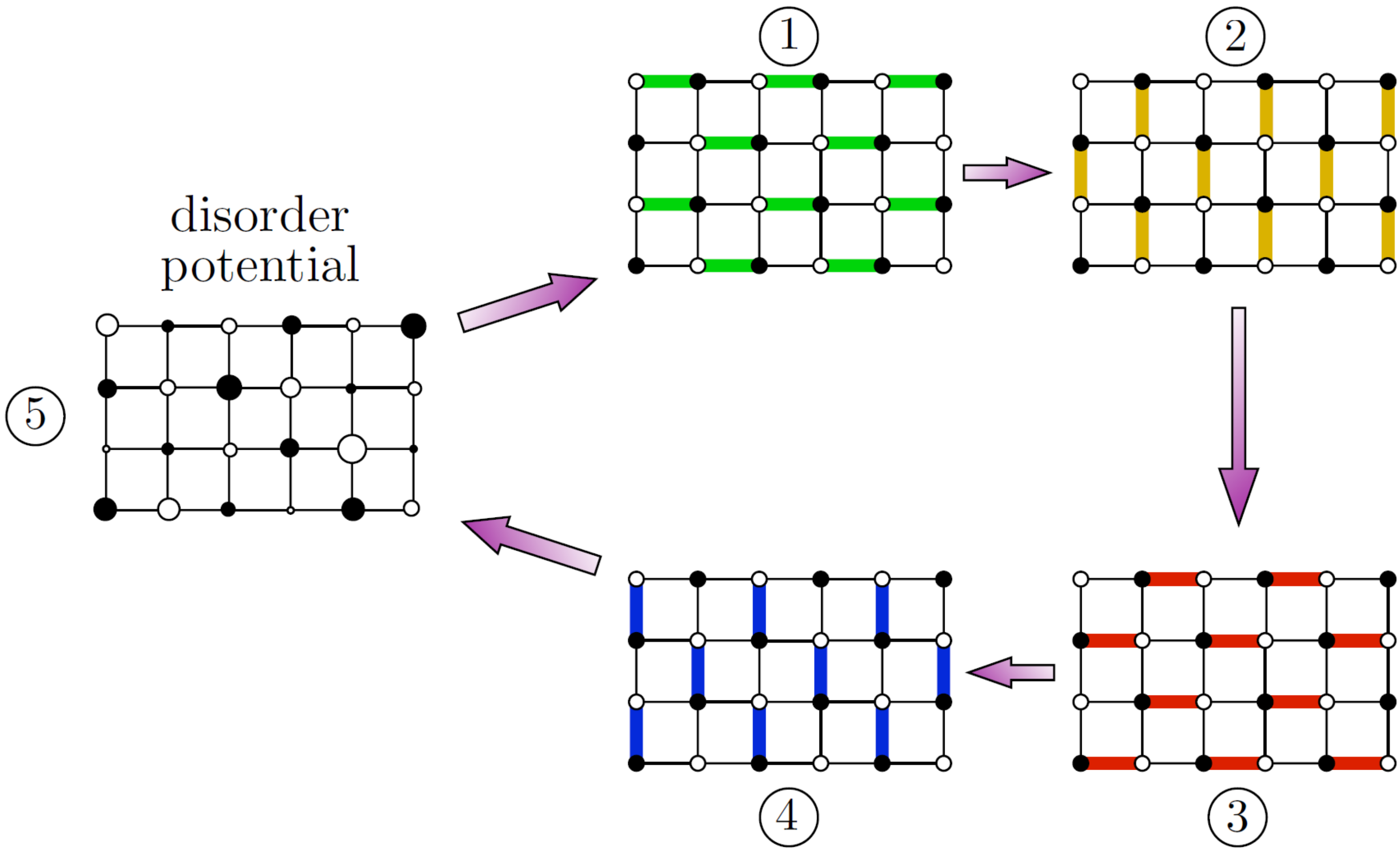


"Hopping pulse" strength $\frac{JT}{4} = \frac{\pi}{2}$

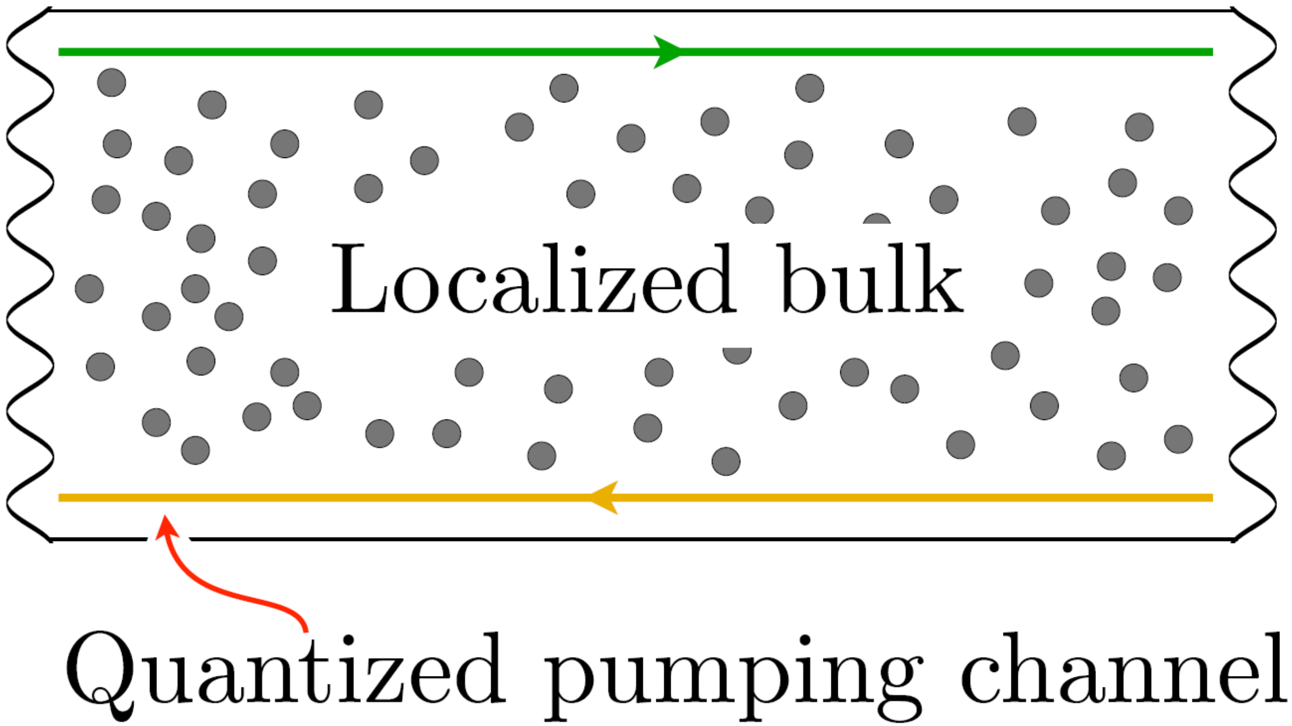
Outline

- **Review**: Floquet, Bloch, Floquet-Bloch
- **Disorder** and the Anomalous Floquet-Anderson Insulator (AFAI)
- **Interactions** and quasi-steady states

Disorder localizes all bulk states

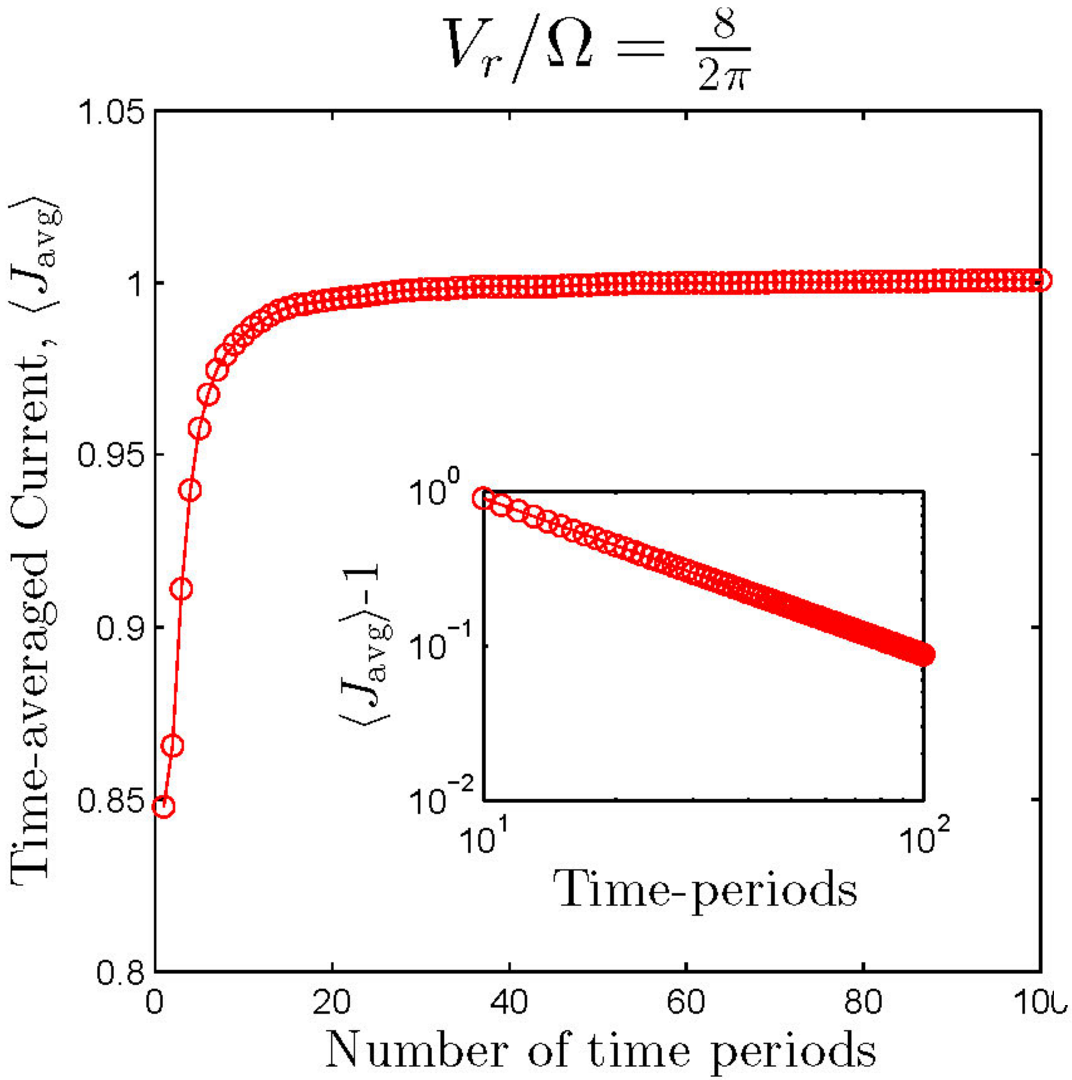


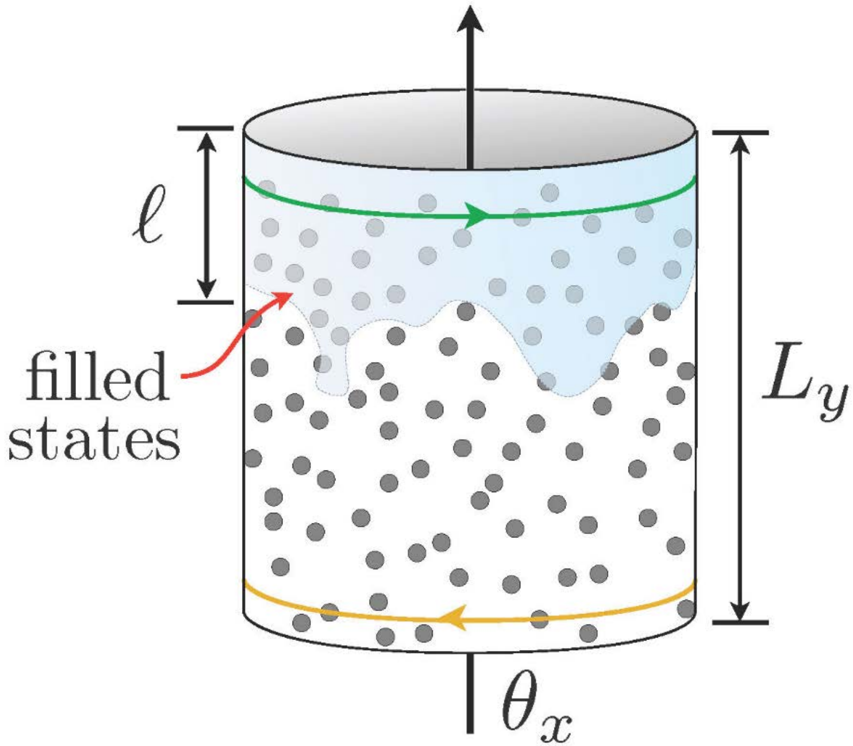
Anomalous Floquet-Anderson Insulator: fully localized bulk with propagating chiral edge states



P. Titum, EB, M. Rudner, G. Refael, and N. Lindner, arXiv:1506.00650

Quantized pumping current in the AFAI

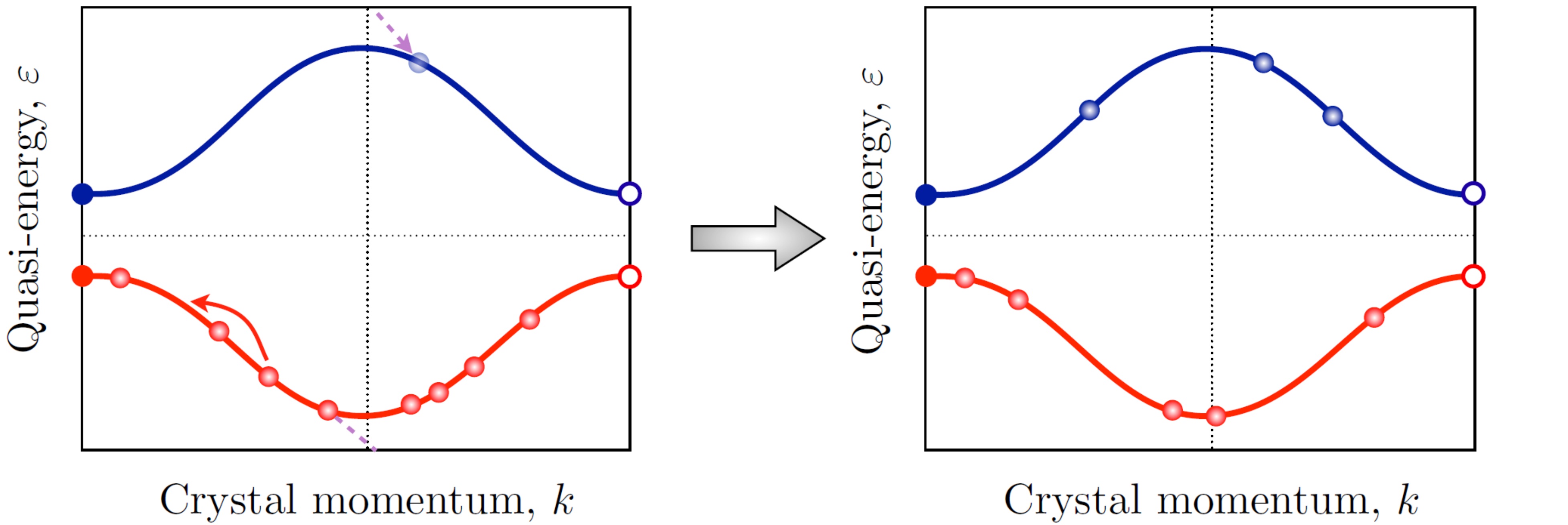




Outline

- **Review**: Floquet, Bloch, Floquet-Bloch
- **Disorder** and the Anomalous Floquet-Anderson Insulator (AFAI)
- **Interactions** and quasi-steady states

At long times, driven interacting (closed) system generically heats to infinite temperature



See for example (plus others):

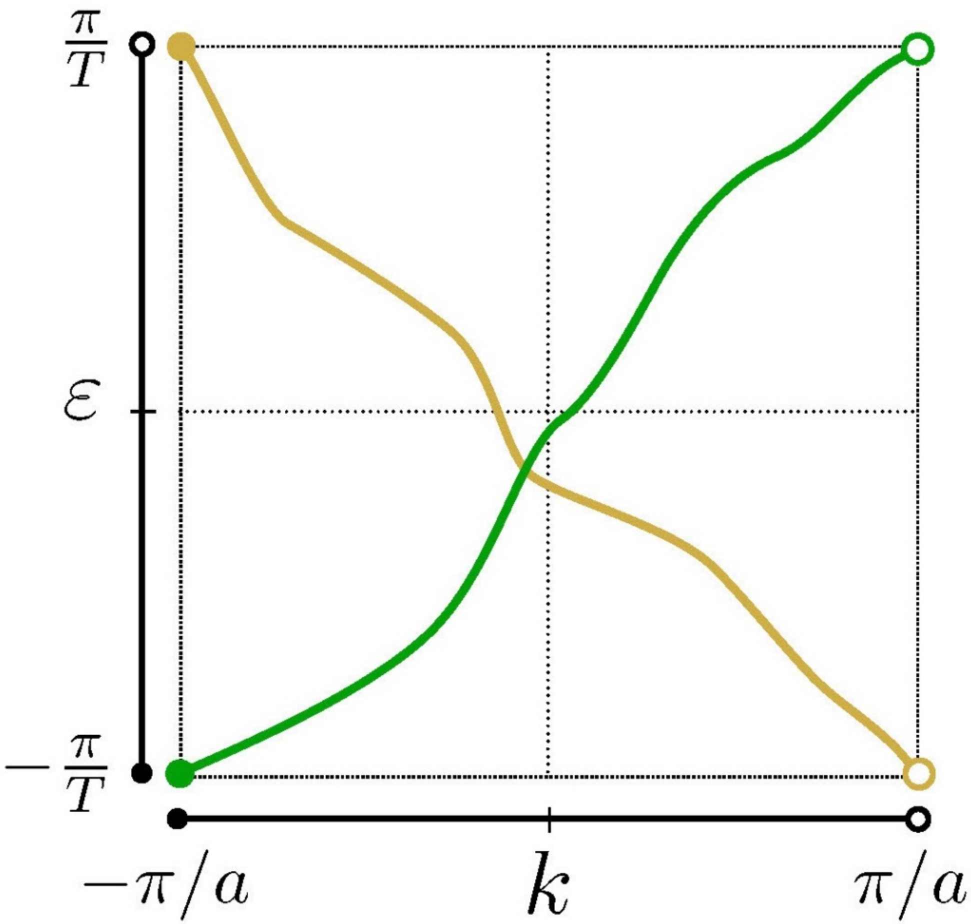
A. Lazarides, A. Das, and R. Moessner, Phys. Rev. Lett. **112**, 150401 (2014).

L. D'Alessio, M. Rigol, arXiv:1402.5141 (2014).

P. Ponte, A. Chandran, Z. Papic, and D. A. Abanin, arXiv:1403:6480 (2014).

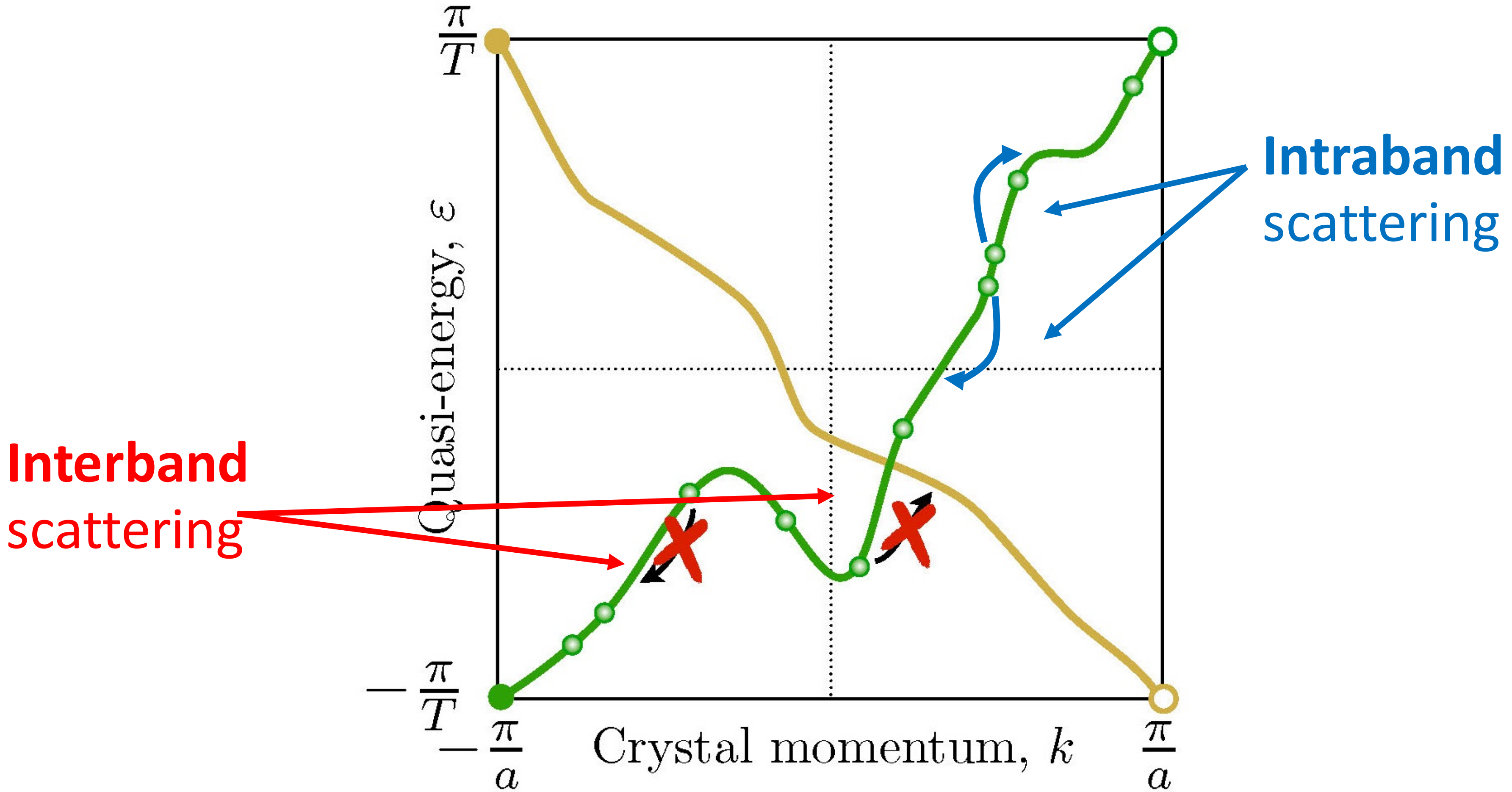
Quasi energy winding

- In Thouless' pump, the two counter propagating bands decouple in the adiabatic limit:

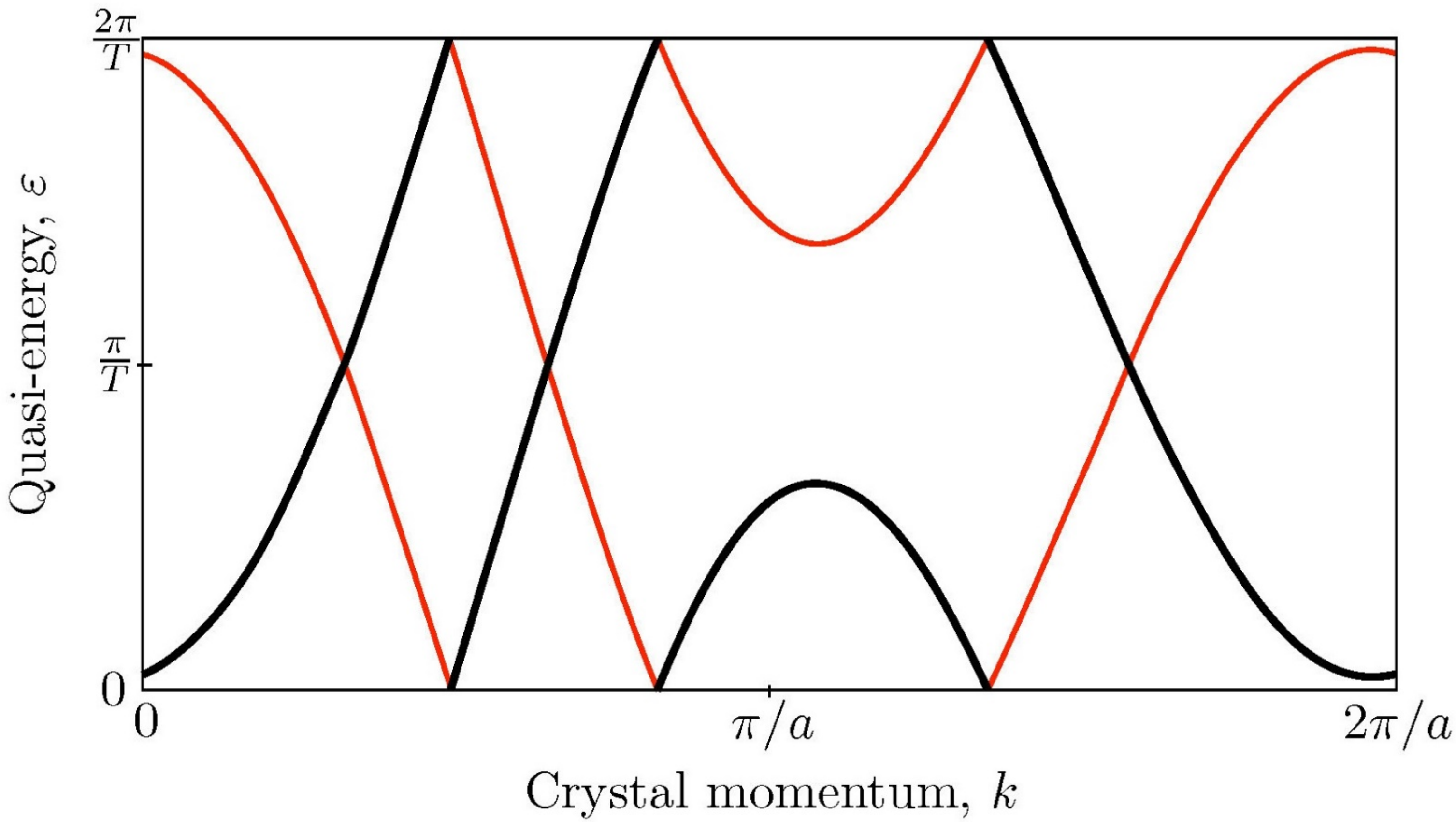


Chiral "thermalization"

- If **interband** scattering is suppressed, particles can ``thermalize'' only within one of the chiral bands

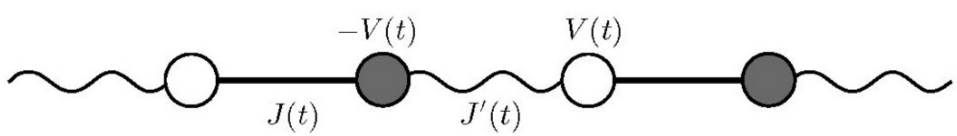


Floquet system with non-trivial winding



$$\frac{V_1}{\hbar\omega} = 8.2$$

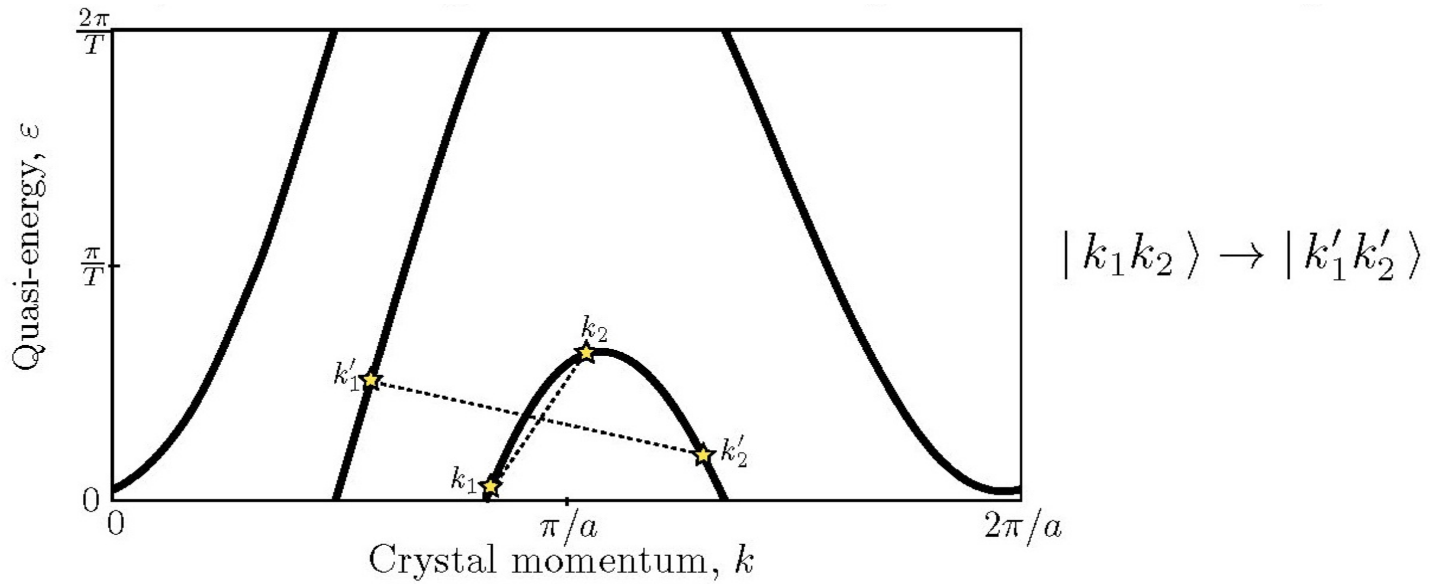
$$\frac{J_0}{\hbar\omega} = 4.1$$

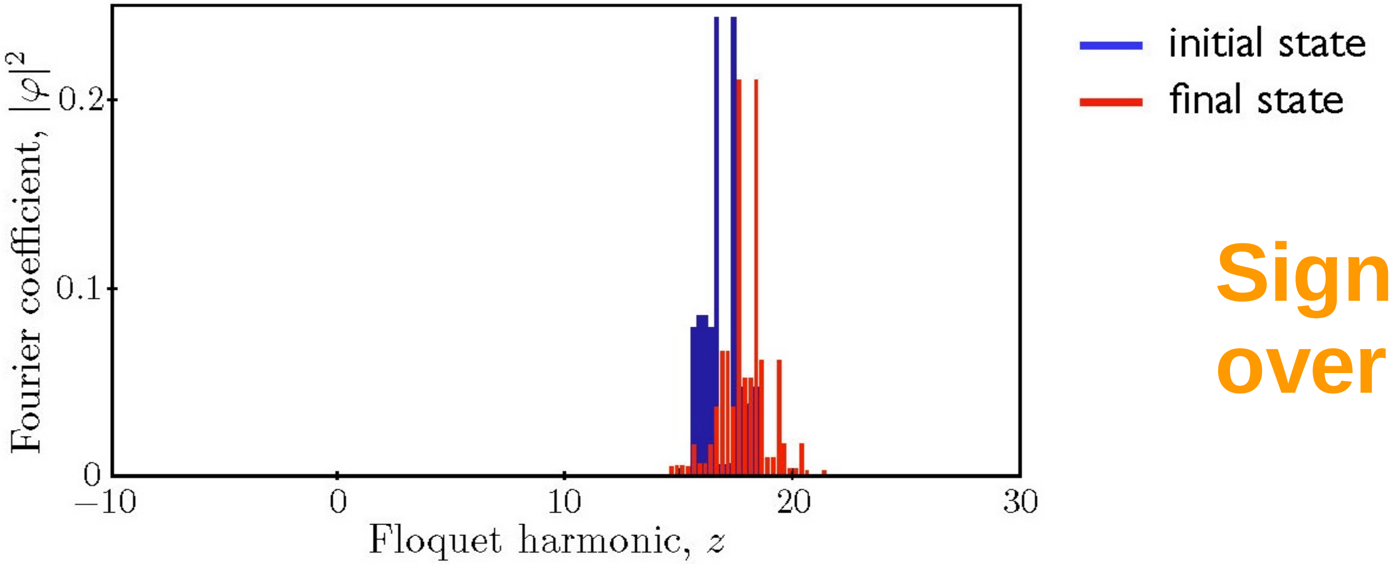


$V(t) = V_1 \sin \omega t \quad J(t) = J_0(1 + \cos \omega t) \quad J'(t) = J_0(1 - \cos \omega t)$

Intraband scattering

Two-particle Floquet state: $|\psi(t)\rangle = e^{-i\varepsilon t}\sum_{z=-\infty}^{\infty} e^{iz\omega t}|\varphi_z\rangle$

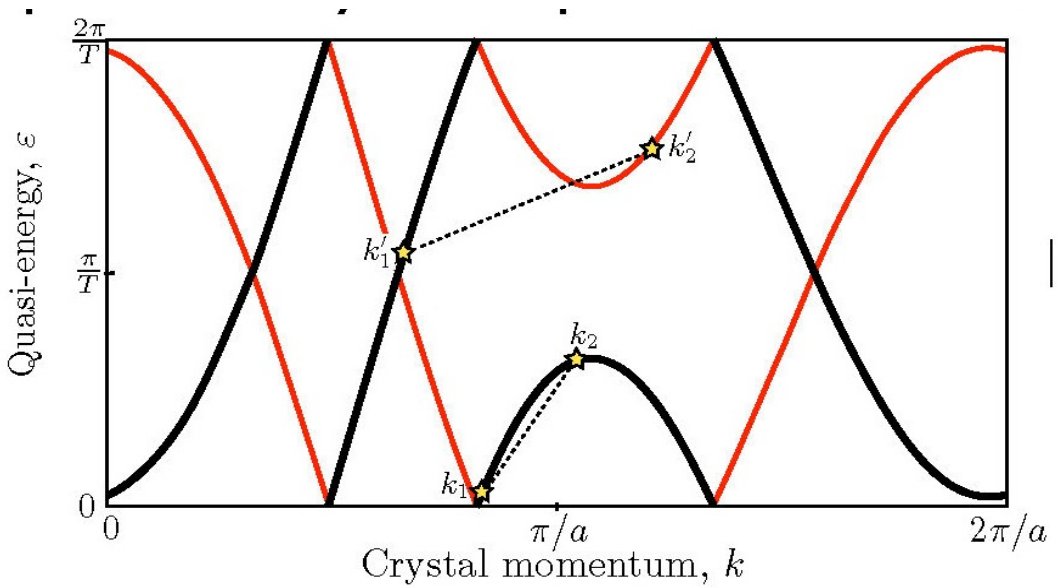




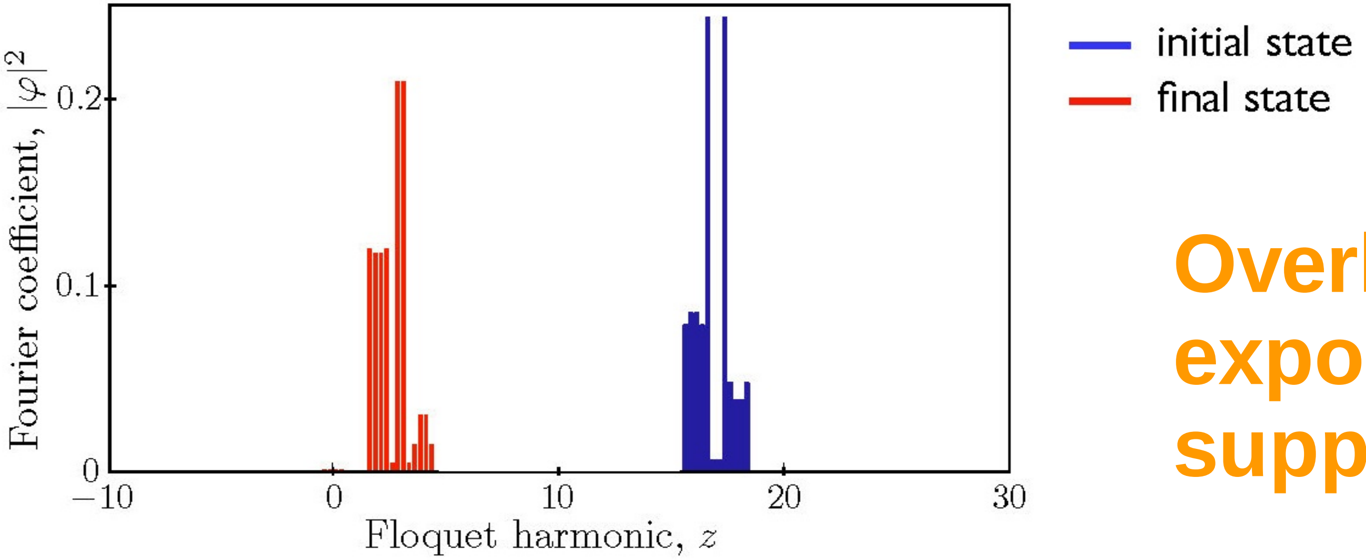
Significant overlap

Interband scattering

Two-particle Floquet state: $|\psi(t)\rangle = e^{-i\varepsilon t}\Sigma_{z=-\infty}^{\infty} e^{iz\omega t}|\varphi_z\rangle$



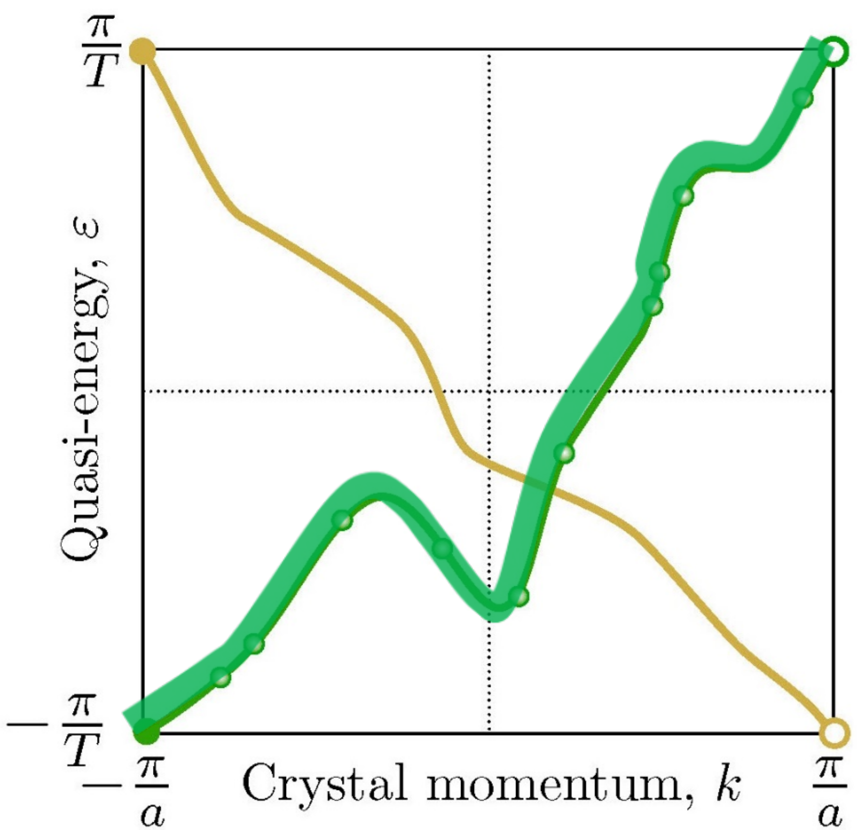
$|\,k_1k_2\,\rangle \rightarrow |\,k_1'k_2'\,\rangle$



Overlap exponentially suppressed

Chiral quasi-steady state

- At intermediate times, a chiral state emerges exhibiting quantized pumping current:



- <u>Initial current:</u> arbitrary
- <u>Intermediate times:</u>

$$t_{\text{intra}} < t < t_{\text{inter}}$$

$$\text{current} = \frac{1}{T} \times \text{density}$$

- Current decays for

$$t > t_{\text{inter}}$$

M. Rudner, N. Lindner and EB, work in progress

Summary

Periodically driven systems can host a variety of topological phenomena, with no analogues in static systems.

- Chiral edge states with no Chern numbers
- **Disorder**: chiral edge states with fully localized bulk, non-adiabatic quantized charge pumping
- **Interactions:** in closed systems, driving generically leads to indefinite heating…
  …But unusual long-lived quasi-steady states are possible

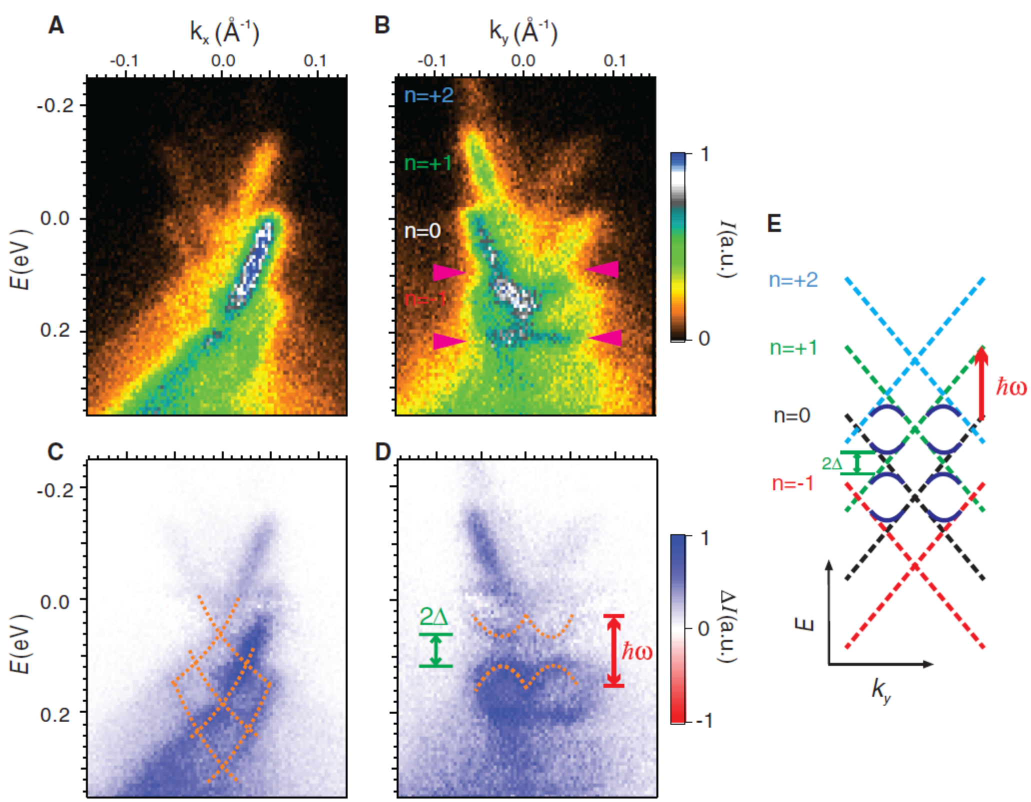
Thank you.

Experiments

Observation of Floquet-Bloch States on the Surface of a Topological Insulator

Y. H. Wang,* H. Steinberg, P. Jarillo-Herrero, N. Gedik†

25 OCTOBER 2013 VOL 342 **SCIENCE** www.sciencemag.org



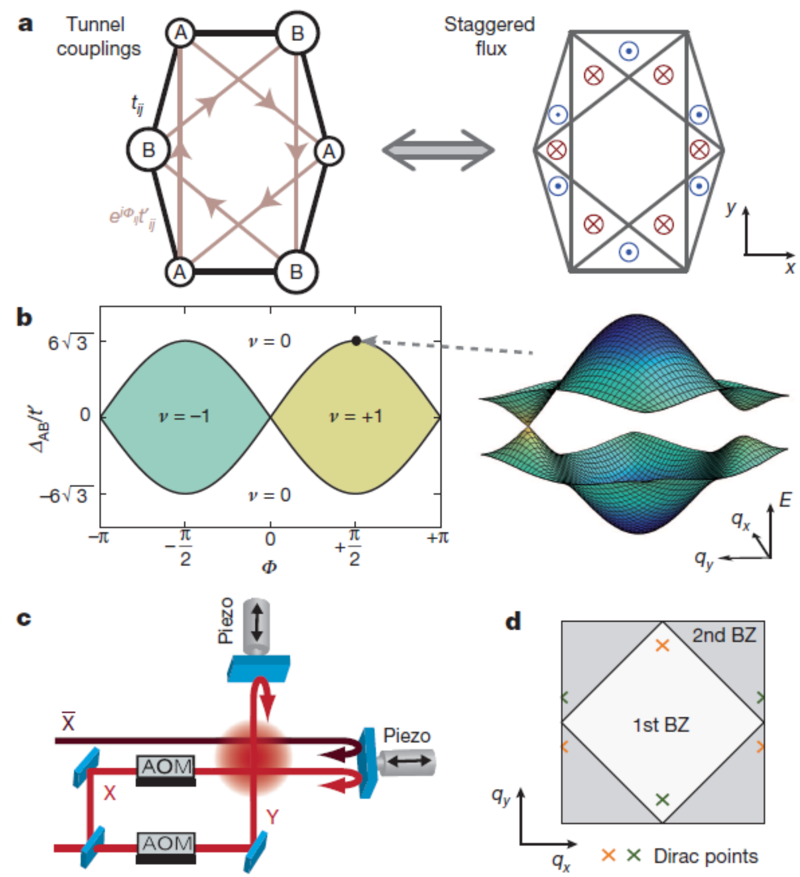
LETTER

doi:10.1038/nature13915

Experimental realization of the topological Haldane model with ultracold fermions

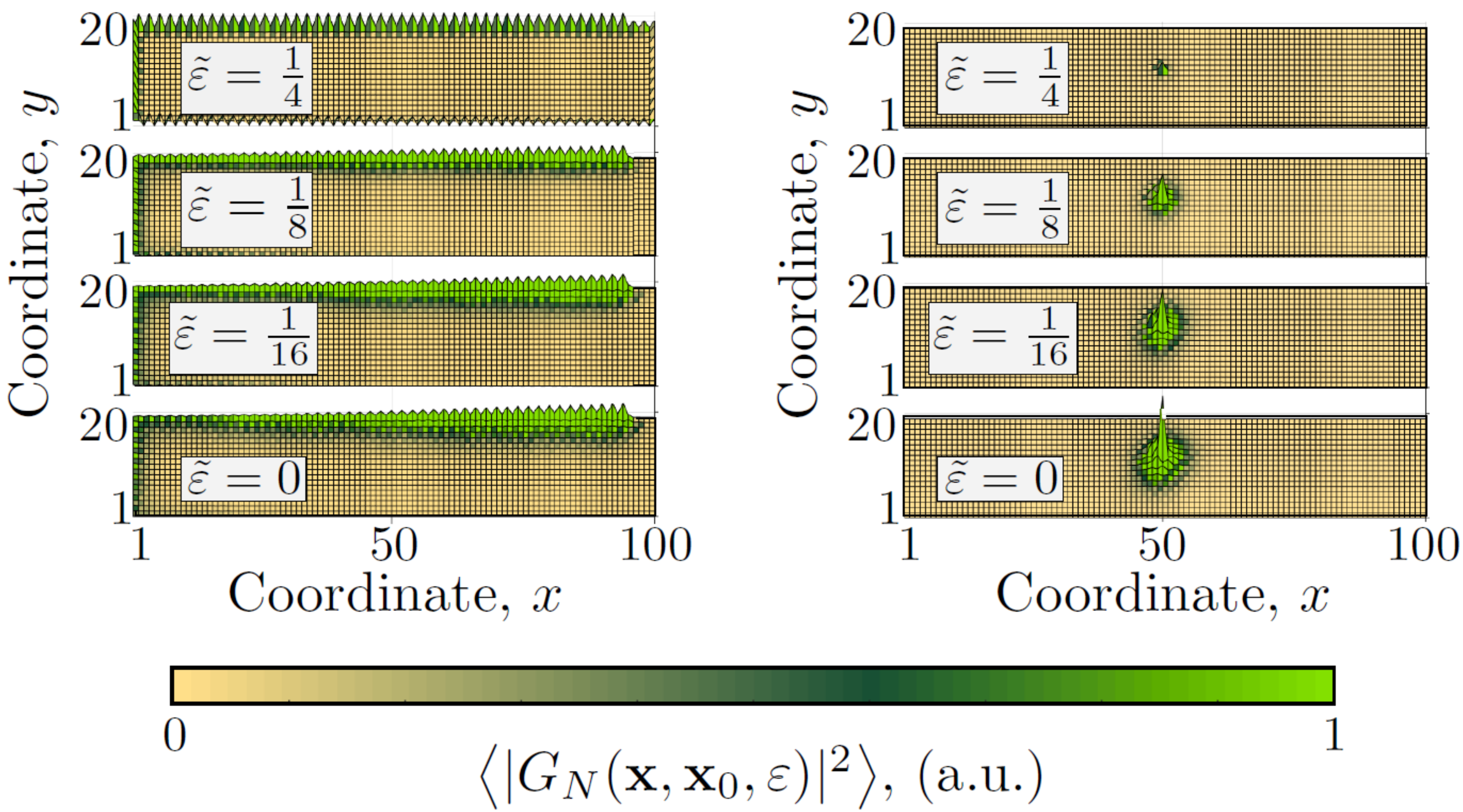
Gregor Jotzu[1], Michael Messer[1], Rémi Desbuquois[1], Martin Lebrat[1], Thomas Uehlinger[1], Daniel Greif[1] & Tilman Esslinger[1]

13 NOVEMBER 2014 | VOL 515 | NATURE | 237



Numerical Simulations

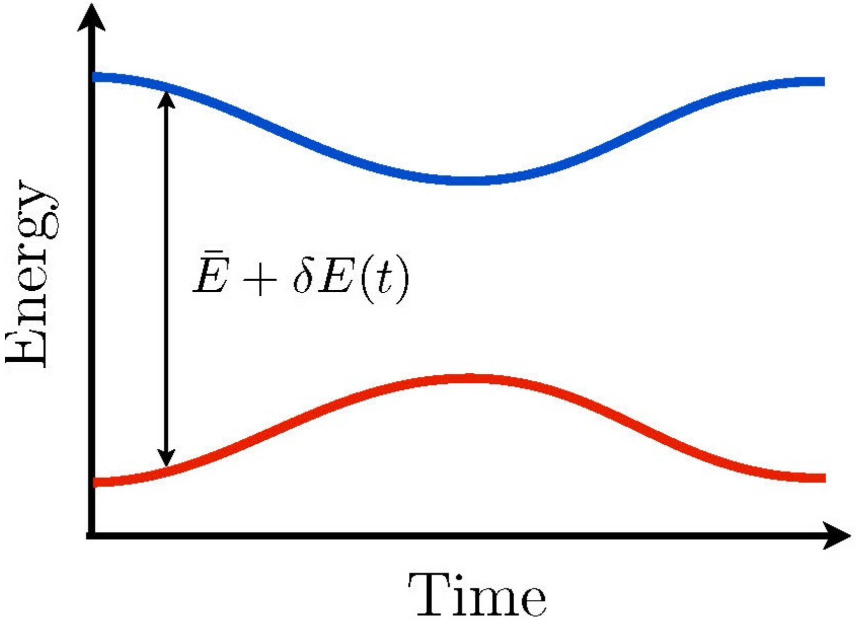
Green's function at different quasi-energies:

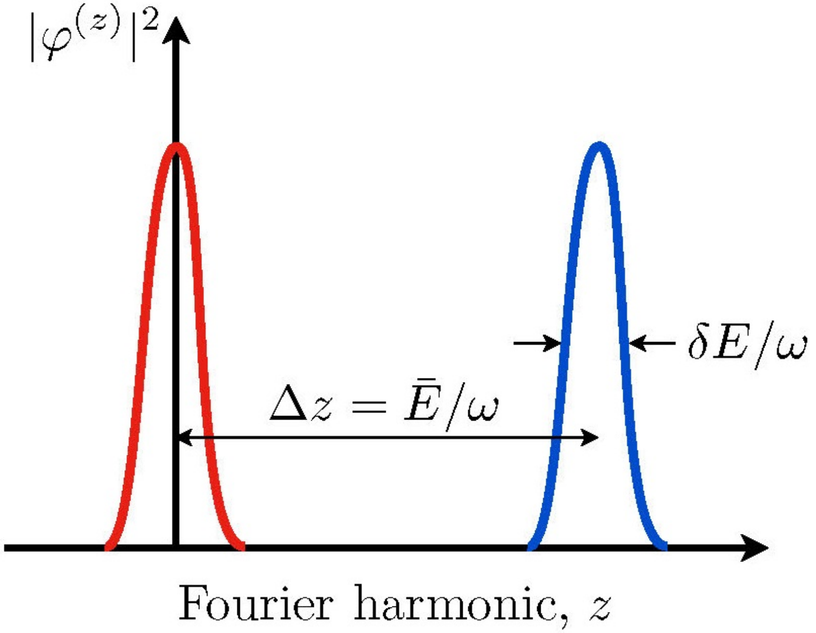


Frequency space analysis

- Matrix element for scattering controlled by the overlap of Fourier components of the scattering Floquet states

$$|\psi(t)\rangle = e^{i\varepsilon t} \sum_{z=-\infty}^{\infty} e^{iz\omega t} |\varphi_z\rangle$$

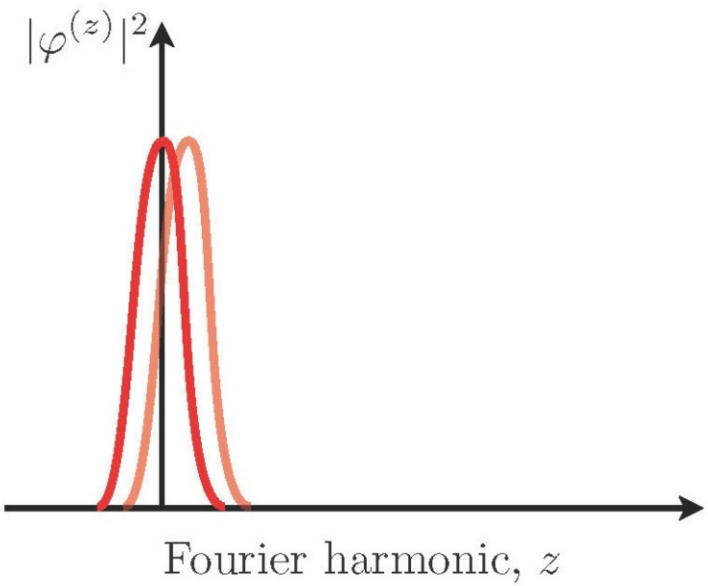




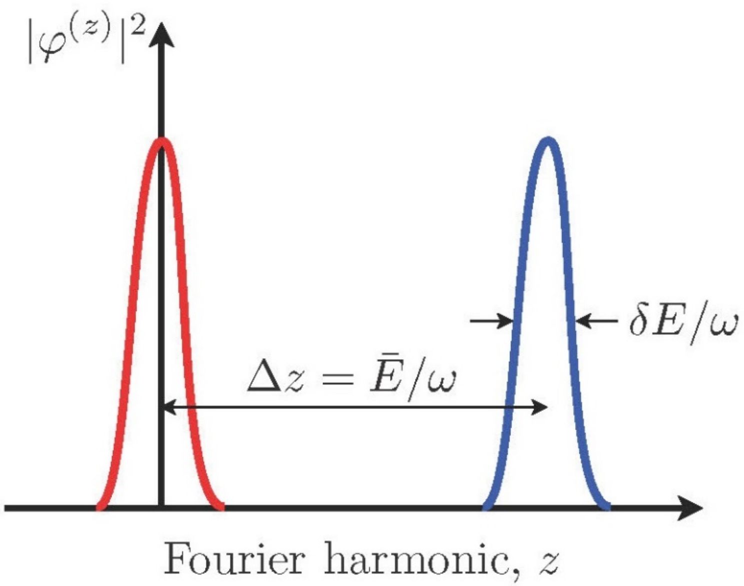
Frequency space analysis

- Matrix element controlled by the overlap of Fourier components of the scattering Floquet states

Intraband scattering

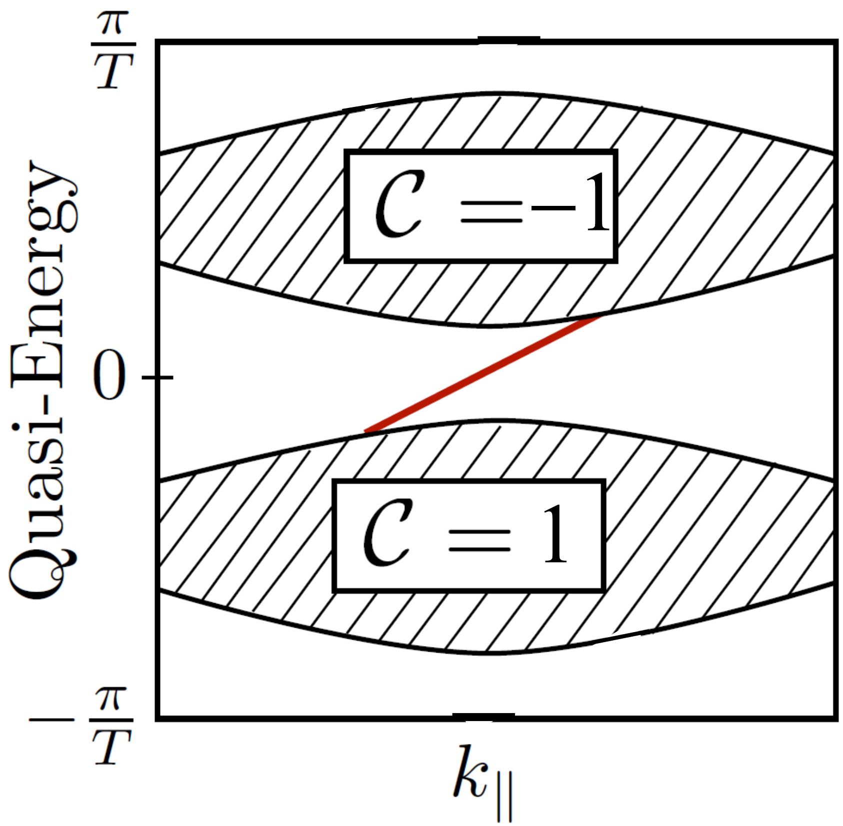


Interband scattering



Two Dimensions: Chern Bands

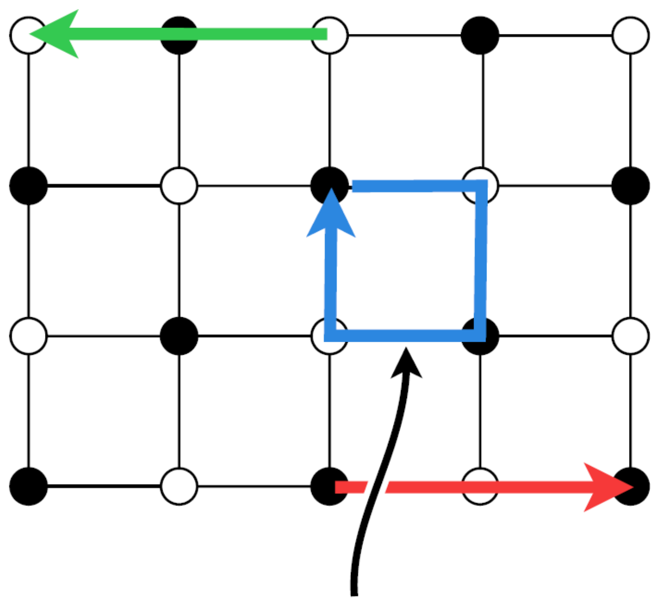
$$C = \frac{1}{\pi} \int d^2k \; \mathrm{Im}\langle \partial_{k_x} \psi_{n\mathbf{k}} | \partial_{k_y} \psi_{n\mathbf{k}} \rangle$$



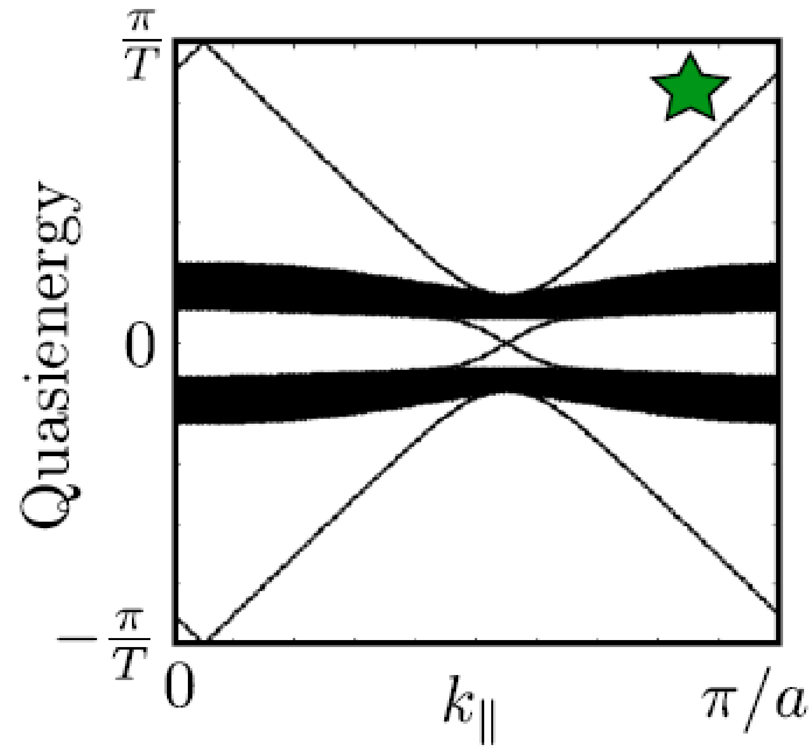
Chern numbers of bulk bands related to chiral edge states

Bands with $C \neq 0$ cannot be fully localized in the presence of disorder

Bulk evolution trivial, chiral modes propagate along edges



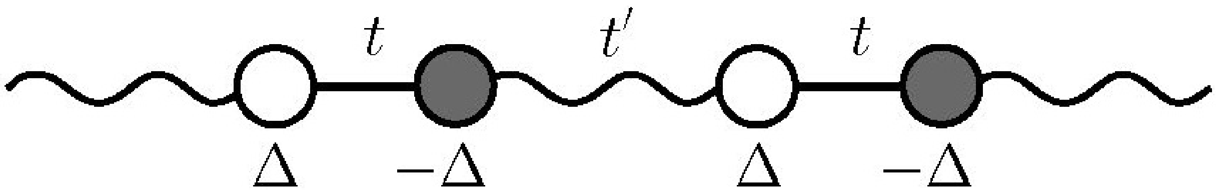
Particle returns to
initial position after cycle



“Hopping pulse” strength $\frac{JT}{4} \neq \frac{\pi}{2}$

Quantized adiabatic charge pumping

Thouless (83')



$$t = t_0 + \delta$$
$$t' = t_0 - \delta$$

How does this pump look in terms of Flouqet-Bloch bands?

Periodically driven quantum systems

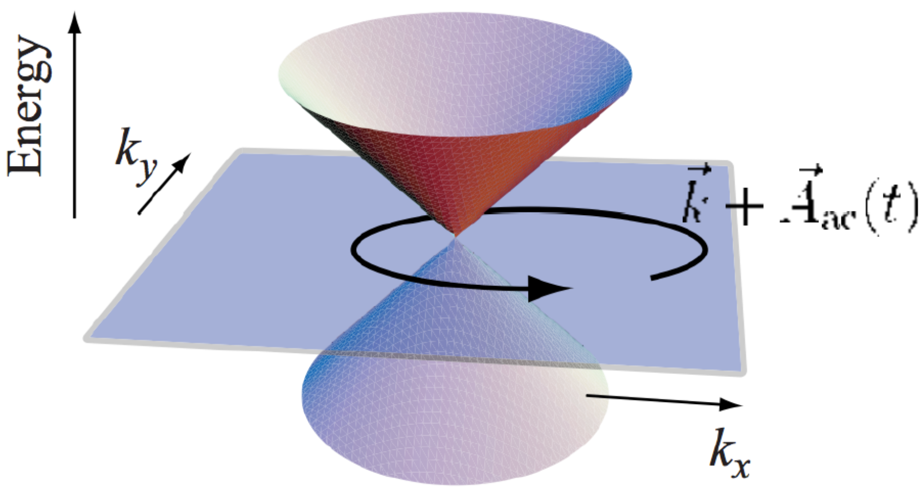
No ground state, energy conservation for driven system

$$i\frac{d}{dt}|\psi\rangle = H(t)|\psi\rangle; \quad H(t+T) = H(t)$$

↑ periodic driving

Optical control of band topology discussed for various setups

Circularly-polarized light opens Haldane gap in graphene



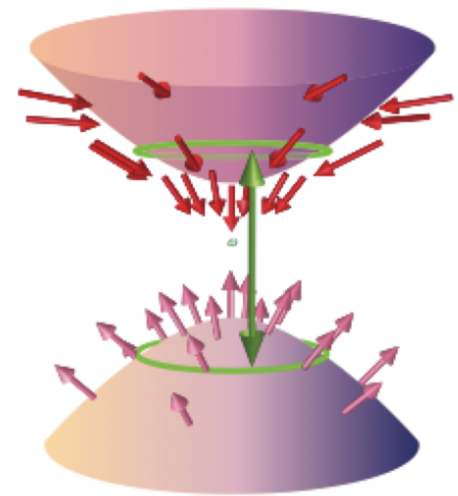
T. Oka and H. Aoki, Phys. Rev. B **79**, 081406 (2009).
T. Kitagawa, et al., Phys. Rev. B **84**, 235108 (2011).
Z. Gu et al., Phys. Rev. Lett. 107, 216601 (2011).

Resonant driving used to create band inversion

ARTICLES
PUBLISHED ONLINE: 13 MARCH 2011 | DOI: 10.1038/NPHYS1926
nature physics

Floquet topological insulator in semiconductor quantum wells

Netanel H. Lindner[1,2]*, Gil Refael[1,2] and Victor Galitski[3,4]



N. Lindner, G. Refael, and V. Galitski, Nature Physics **7**, 490 (2011).