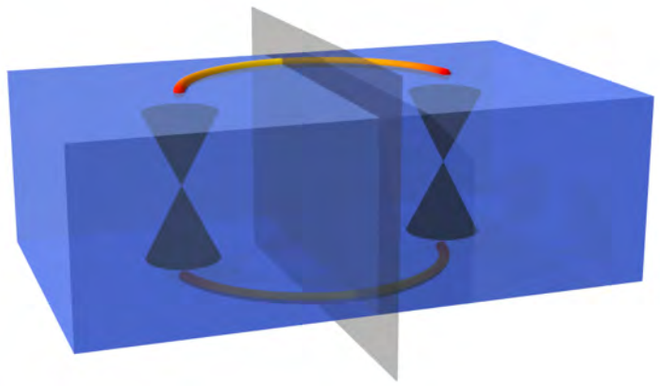
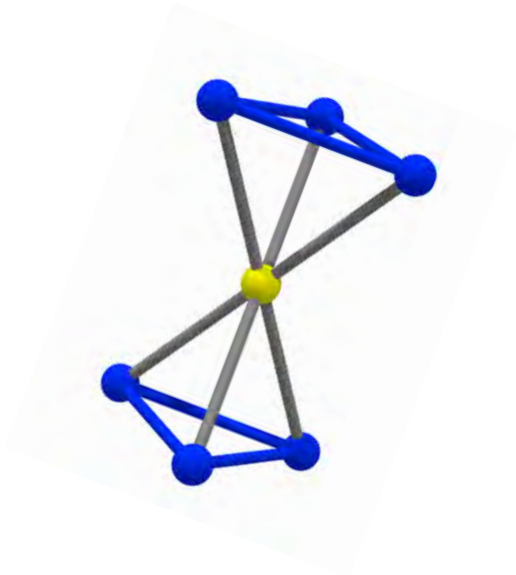
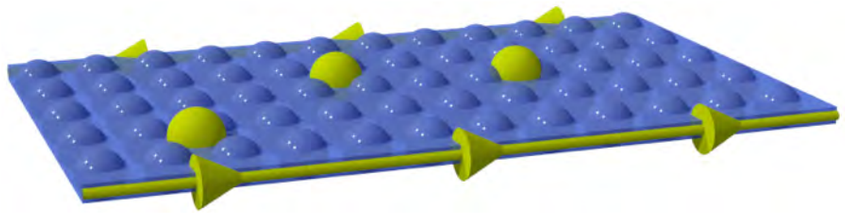




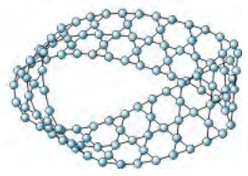
Tuning between Weyl semimetals and fractional Chern insulators in frustrated materials

Emil J. Bergholtz
FU Berlin

Freie Universität Berlin

Emmy Noether-Programm
Deutsche Forschungsgemeinschaft DFG

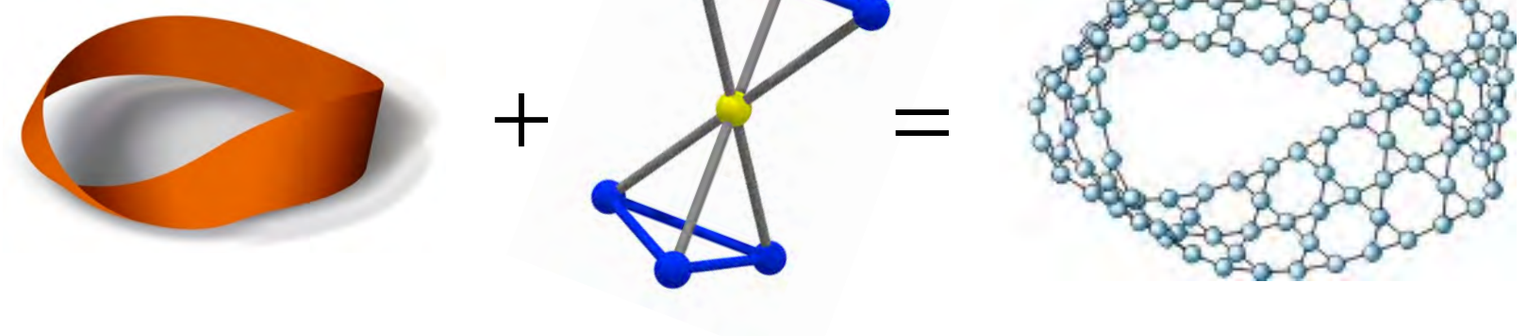
HELMHOLTZ ASSOCIATION
Virtual Institute: New States of Matter and their Excitations



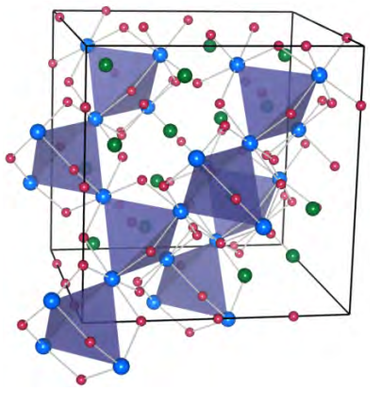
LDQMC workshop, Amsterdam, July 2, 2015

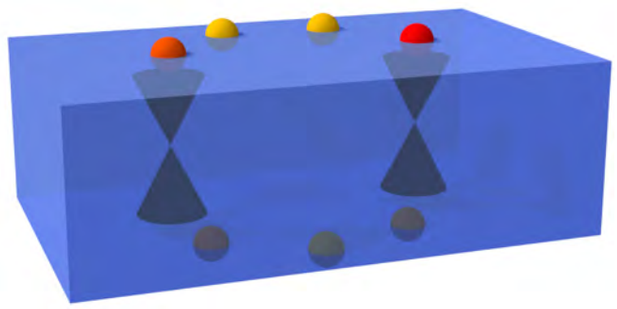
In this talk, I will

- Combine geometrical frustration and band topology



- Argue that *frustrated systems* naturally encompass novel fractional Chern insulators as well as Fermi arcs and Weyl semimetals





- Briefly tell you about a novel and controversial phase transition

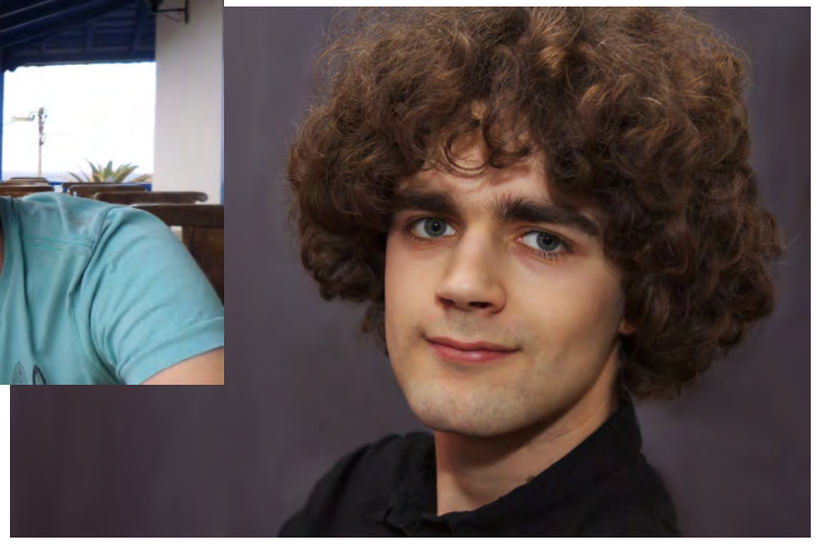


First: My collaborators

- In Berlin

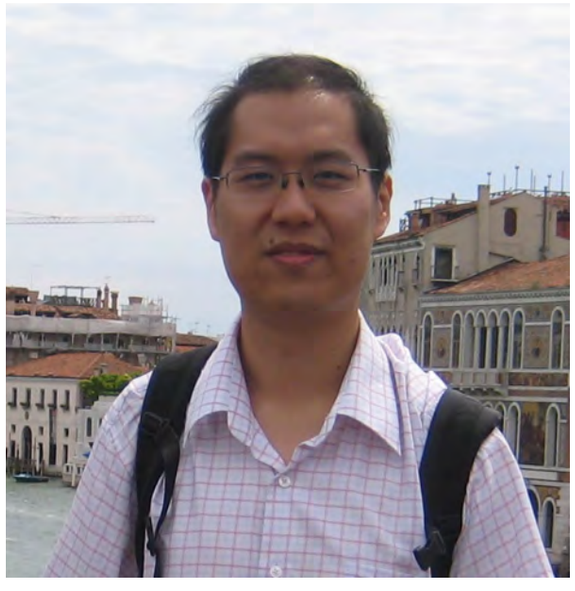
  Jörg Behrmann
  Piet Brouwer
  Flore Kunst
  Gregor Pohl
  Björn Sbierski
  Maximilian Trescher





- External

  Zhao Liu, Princeton -> Berlin
  Andreas Läuchli, Innsbruck
  Roderich Moessner, Dresden
  Masafumi Udagawa, Tokyo



Materials inspiration: Oxide interfaces & geometrically frustrated systems with strong spin-orbit coupling

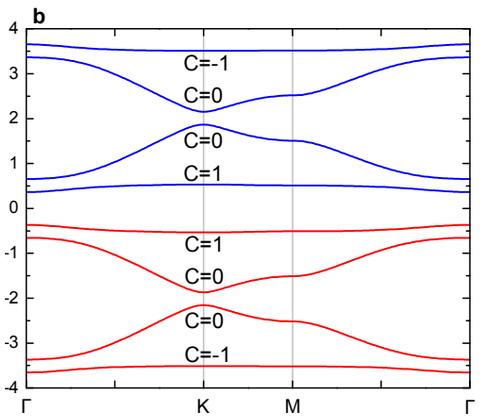
- Perovskite materials, ABO_3, routinely grown in sandwich structures in the [100] direction

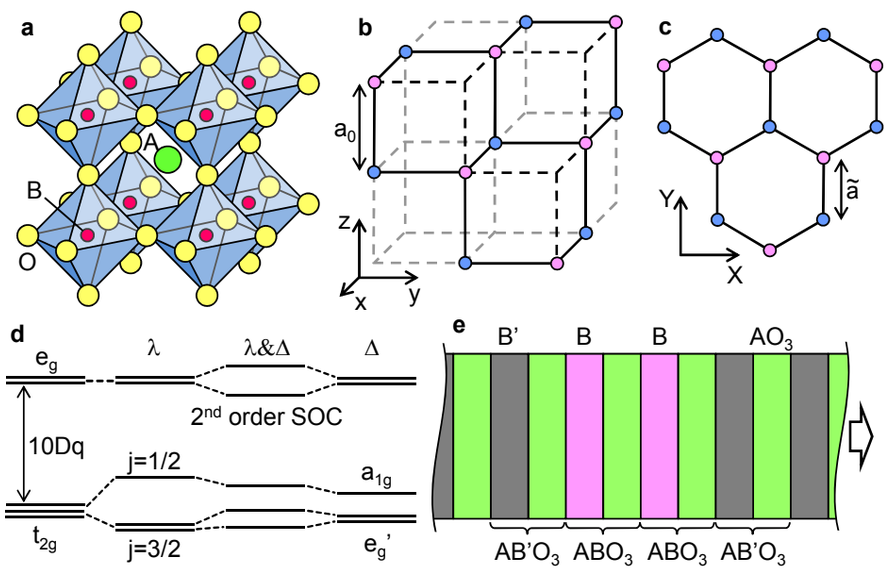
  - Instead (111) slabs would be good for topological physics (relatively flat C=1 bands).

Epitaxial growth of (111)-oriented $LaAlO_3/LaNiO_3$ ultra-thin superlattices

S. Middey,[1, a)] D. Meyers,[1] M. Kareev,[1] E. J. Moon,[1] B. A. Gray,[1] X. Liu,[1] J. W. Freeland,[2] and J. Chakhalian[1]
[1)] *Department of Physics, University of Arkansas, Fayetteville, Arkansas 72701, USA*
[2)] *Advanced Photon Source, Argonne National Laboratory, Argonne, Illinois 60439, USA*

arXiv:1212.0590v1





- Fractional Chern insulators!?
- But [111] is not a natural cleavage/growth direction...

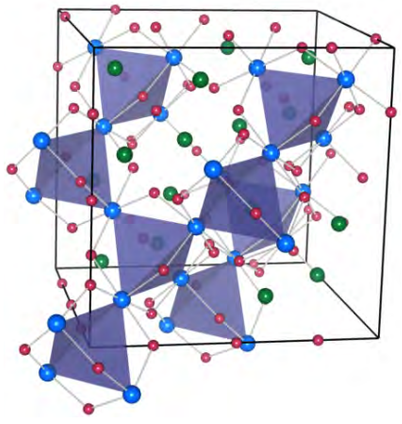
D. Xiao, W. Zhu, Y. Ran, N. Nagaosa, and S. Okamoto, Nature Commun. **2**, 596 (2011).

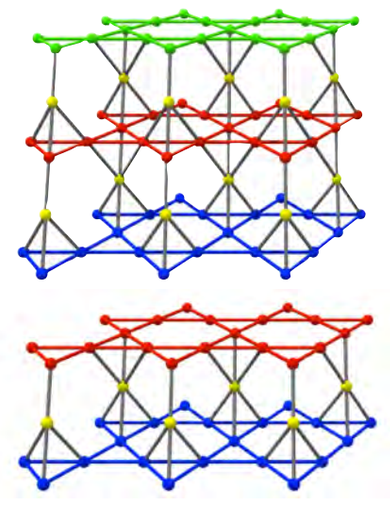
- **Suggestion:** *Consider (111) slabs of pyrochlore transition metal oxides, in particular* $A_2Ir_2O_7$ *iridate thin films*

  - Natural cleavage/growth direction!

  - Provides useful general insights

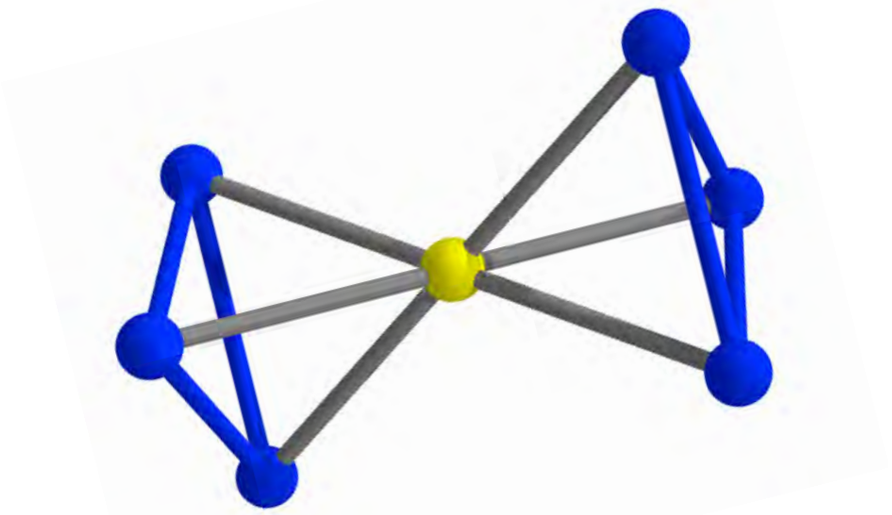
  - Even richer physics…





M. Trescher and E.J. Bergholtz, Phys. Rev. B 86, 241111(R) (2012)

E.J. Bergholtz, Z. Liu, M. Trescher, R. Moessner, and M. Udagawa, Phys. Rev. Lett. 114, 016806 (2015)



Localized modes on frustrated lattices

References:

D. L. Bergman, C. Wu, and L. Balents
Band touching from real space topology in frustrated hopping models
Phys. Rev. B 78, 125104 (2008)

M. Trescher and E.J. Bergholtz,
Flat bands with higher Chern number in pyrochlore slabs
Phys. Rev. B 86, 241111(R) (2012)

E.J. Bergholtz, Z. Liu, M. Trescher, R. Moessner, and M. Udagawa,
Topology and Interactions in a Frustrated Slab: Tuning from Weyl Semimetals to C > 1 Fractional Chern Insulators
Phys. Rev. Lett. 114, 016806 (2015)

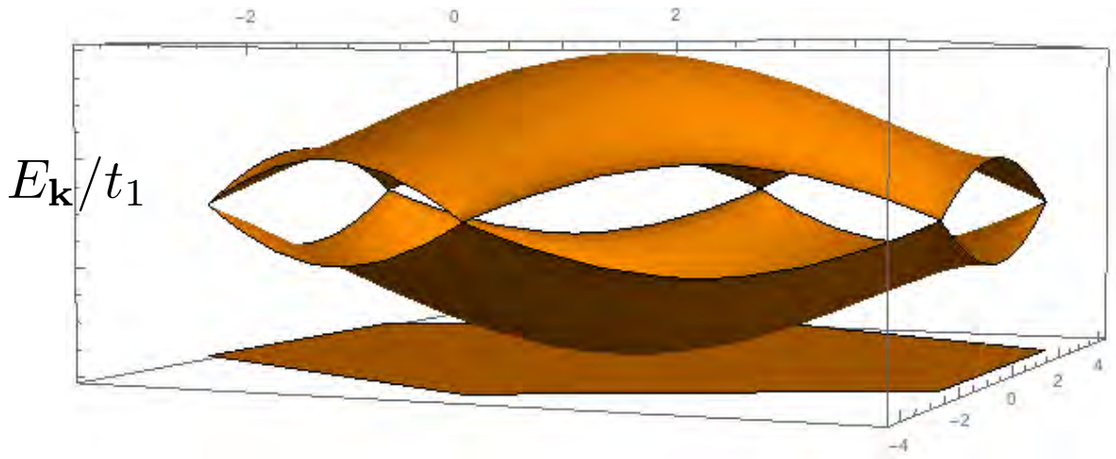
Flat bands and localized modes on frustrated lattices

See e.g.,
D. L. Bergman, C. Wu, and L. Balents
Phys. Rev. B 78, 125104 (2008)

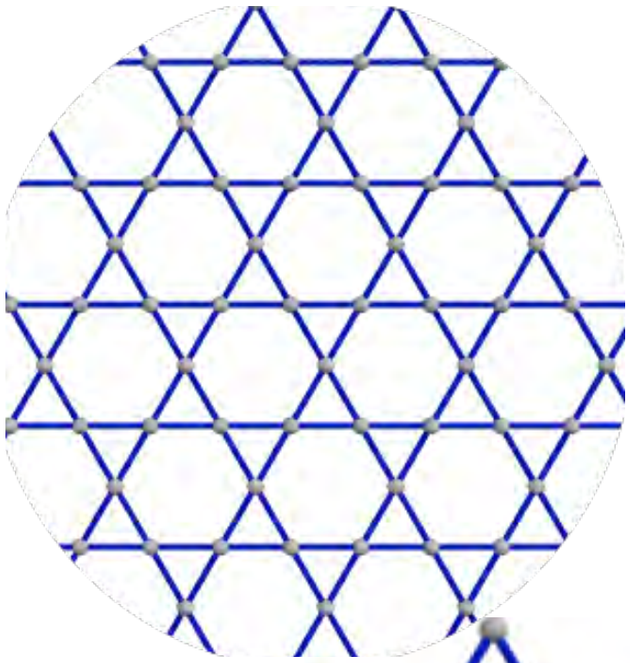
- Example: nearest neighbor hopping on a kagome lattice

$$H = t_1 \sum_{\langle i,j \rangle} c_i^\dagger c_j$$

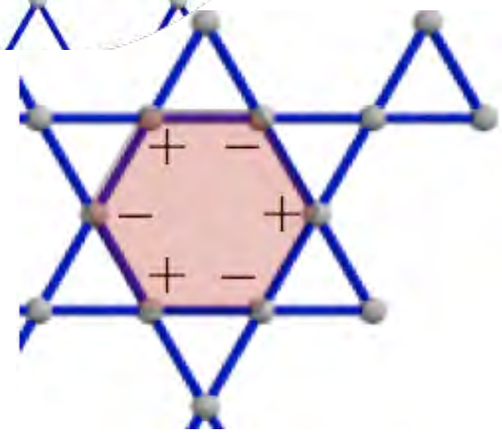
Bloch Hamiltonian: $\mathcal{H}_{\mathbf{k}} = t_1 \begin{pmatrix} 0 & 1+e^{ik_1} & 1+e^{ik_2} \\ 1+e^{-ik_1} & 0 & 1+e^{-ik_3} \\ 1+e^{-ik_2} & 1+e^{ik_3} & 0 \end{pmatrix}$



"Graphene + a flat band"



Localized modes
explain the flat band



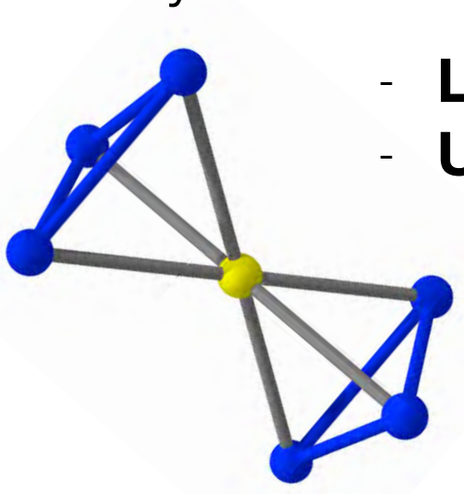
- But these states are neither topological nor Wannier functions!
  - Quadratic touching point
  - Look for a slightly refined concept...

Idea: attempt to localize in the third dimension

M. Trescher and E.J. Bergholtz,
Phys. Rev. B 86, 241111(R) (2012)

- Consider frustrated layered systems, e.g. [111]-grown pyrochlore with kagome layers connected via local hopping to the intermediate on triangular layers

- Crucial insight: surface bands localized to the kagome layers iff the total hopping amplitude to the triangular layer vanish.

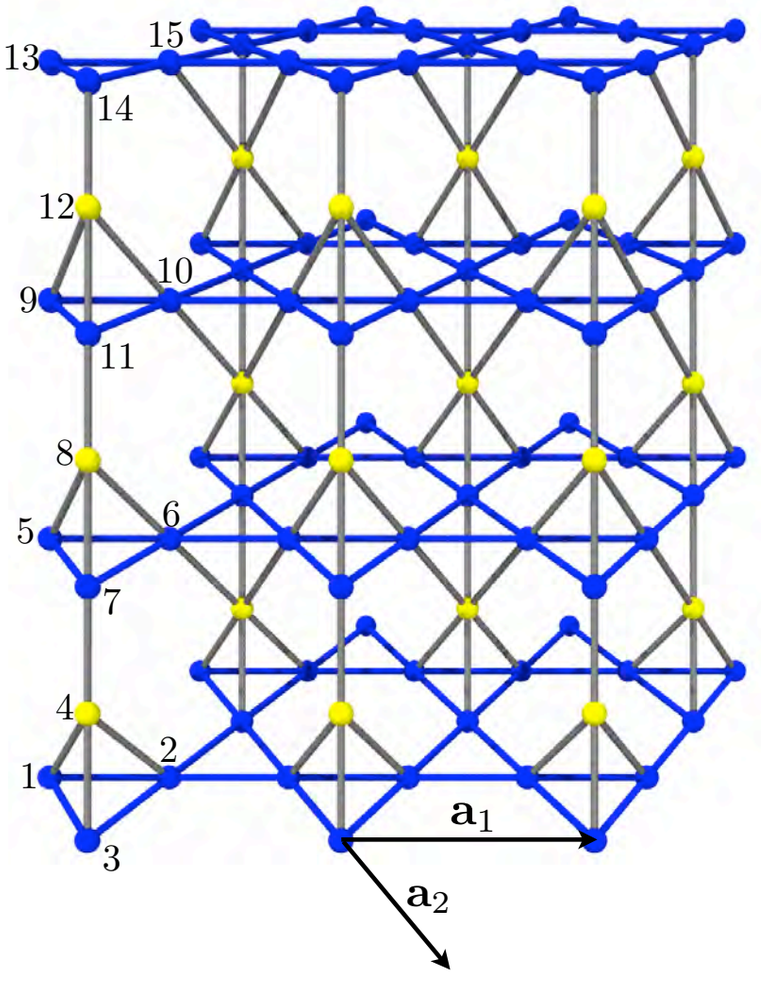


- **Local constraint**, destructive interference
- **Unique solution**, independent of details!

$$|\psi^i(\mathbf{k})\rangle = \mathcal{N}(\mathbf{k}) \sum_{m=1}^{N} \Big(r(\mathbf{k})\Big)^m |\phi^i(\mathbf{k})\rangle_m$$

$$r(\mathbf{k}) = -\frac{\phi_1^i(\mathbf{k}) + \phi_2^i(\mathbf{k}) + \phi_3^i(\mathbf{k})}{e^{-ik_2}\phi_1^i(\mathbf{k}) + e^{i(k_1-k_2)}\phi_2^i(\mathbf{k}) + \phi_3^i(\mathbf{k})}$$

components of the single-layer Bloch spinor

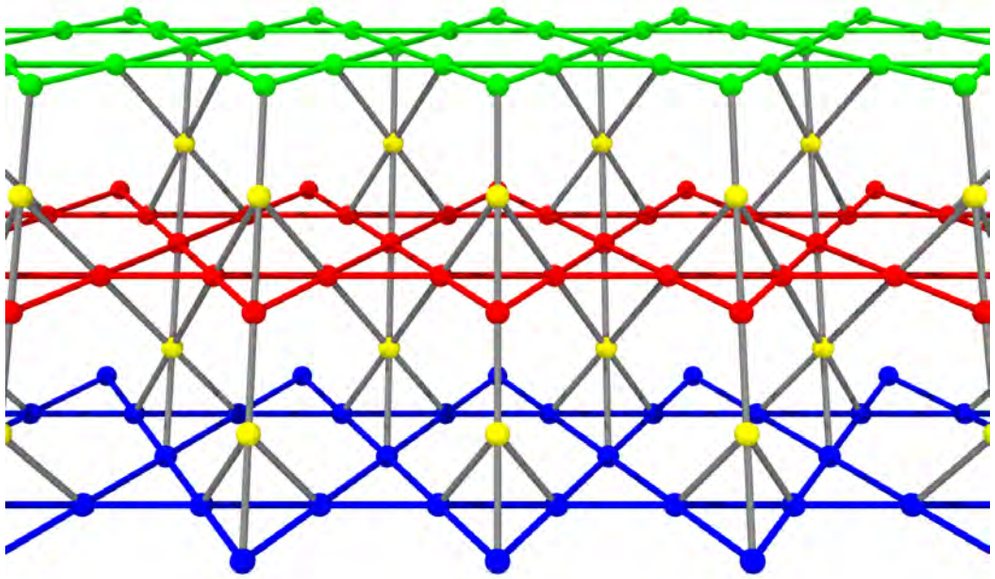


- Inherits the dispersion of the single layer model
- Localized to top or bottom layer, depending on $|r(\mathbf{k})|$
- Reminiscent of Fermi arcs…..

Illuminating, in color…

M. Trescher and E.J. Bergholtz,
Phys. Rev. B 86, 241111(R) (2012)

$|r(\mathbf{k})| > 1$ state localized to the top



$|r(\mathbf{k})| < 1$ state localized to the bottom

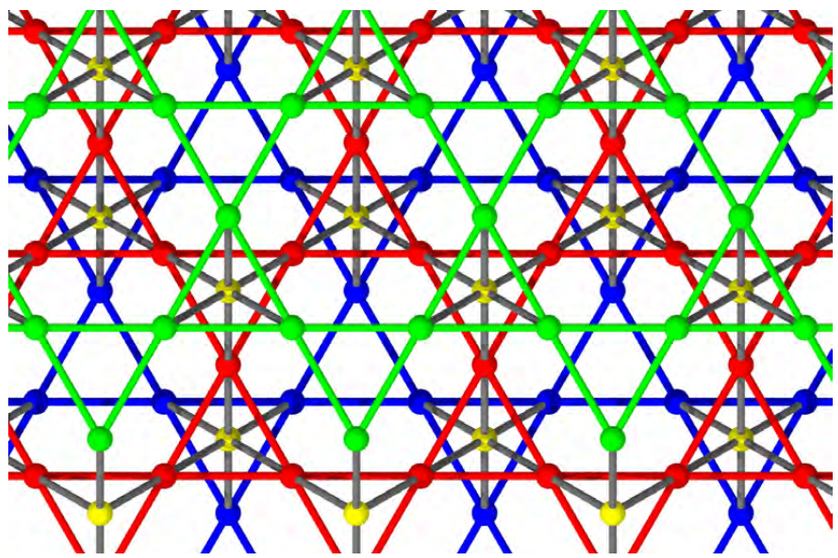
$|r(\mathbf{k})| = 1$ state delocalized!

$$\Psi(\mathbf{k}) = \mathcal{N}(\mathbf{k}) \begin{pmatrix} r^2(\mathbf{k})\phi_1(\mathbf{k}) \\ r^2(\mathbf{k})\phi_2(\mathbf{k}) \\ r^2(\mathbf{k})\phi_3(\mathbf{k}) \\ 0 \\ r(\mathbf{k})\phi_1(\mathbf{k}) \\ r(\mathbf{k})\phi_2(\mathbf{k}) \\ r(\mathbf{k})\phi_3(\mathbf{k}) \\ 0 \\ \phi_1(\mathbf{k}) \\ \phi_2(\mathbf{k}) \\ \phi_3(\mathbf{k}) \end{pmatrix}$$

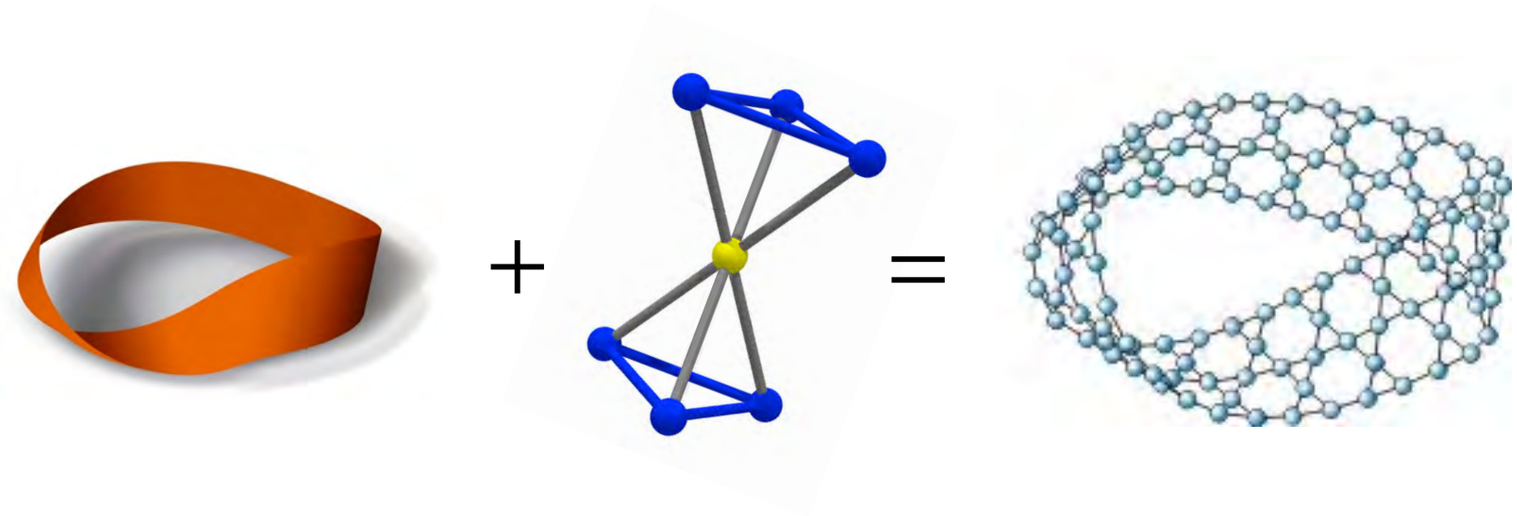
- Non-trivial $r(\mathbf{k})$ due to the **twisted layer structure**

$$r(\mathbf{k}) = -\frac{\phi_1^i(\mathbf{k}) + \phi_2^i(\mathbf{k}) + \phi_3^i(\mathbf{k})}{e^{-ik_2}\phi_1^i(\mathbf{k}) + e^{i(k_1-k_2)}\phi_2^i(\mathbf{k}) + \phi_3^i(\mathbf{k})}$$

But $|r(\mathbf{k})| = 1$ in absence of spin-orbit coupling…



top view



Topology meets frustration

References:

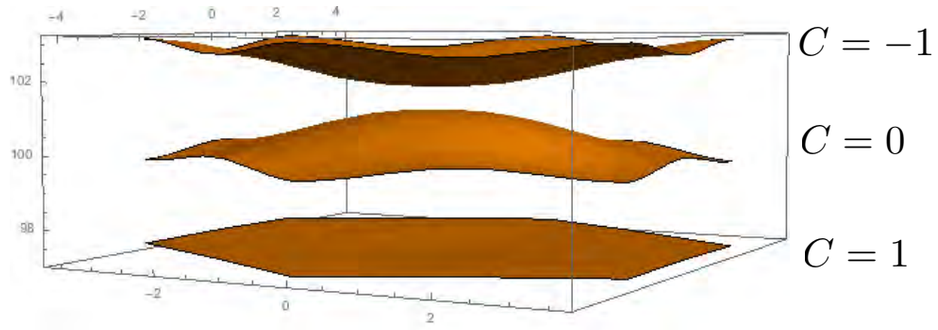
M. Trescher and E.J. Bergholtz,
Flat bands with higher Chern number in pyrochlore slabs
Phys. Rev. B 86, 241111(R) (2012)

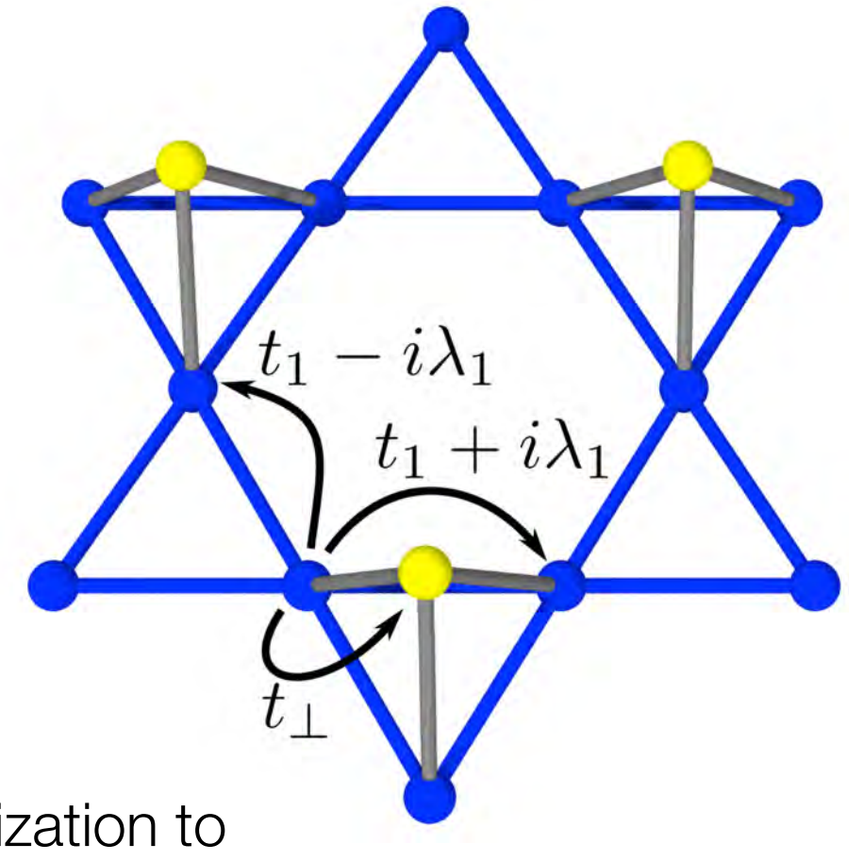
Z. Liu, E.J. Bergholtz, H. Fan, and A. M. Läuchli,
Fractional Chern Insulators in Topological Flat bands with Higher Chern Number
Phys. Rev. Lett. 109, 186805 (2012)

E.J. Bergholtz, Z. Liu, M. Trescher, R. Moessner, and M. Udagawa,
Topology and Interactions in a Frustrated Slab: Tuning from Weyl Semimetals to C > 1 Fractional Chern Insulators
Phys. Rev. Lett. 114, 016806 (2015)

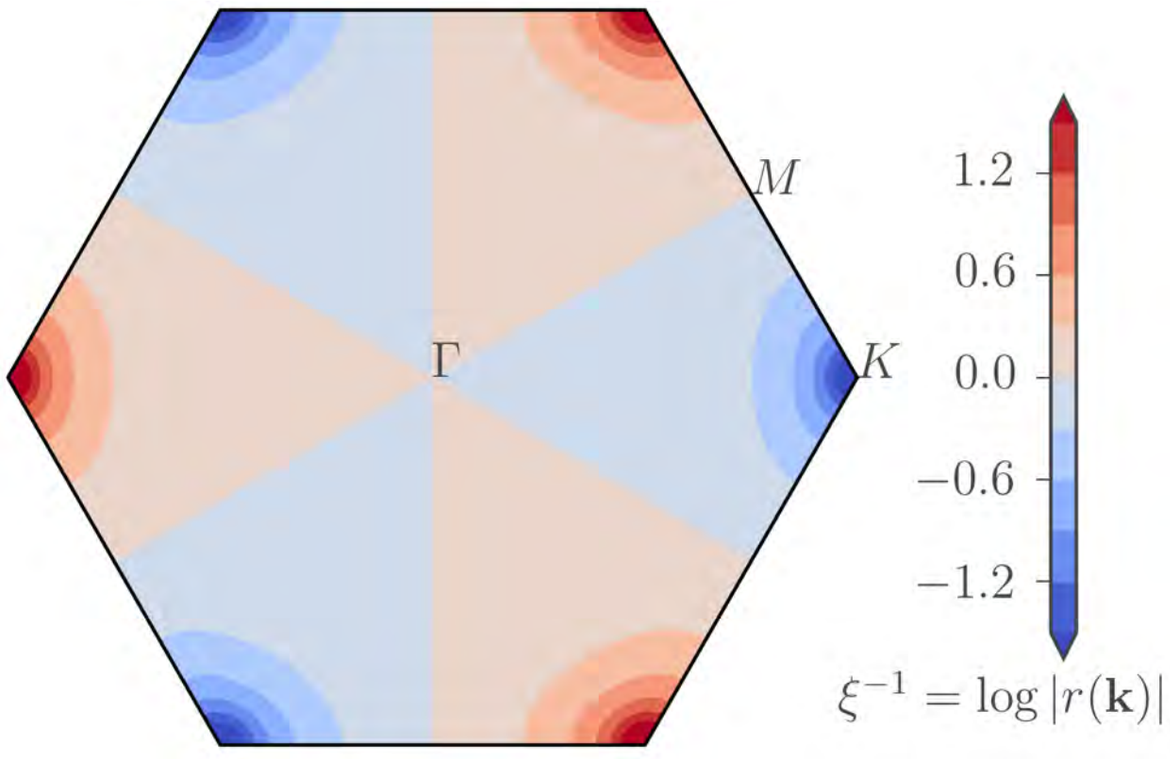
Add spin-orbit coupling

- Each separate layer becomes a Chern insulator





- $|r(\mathbf{k})|$ becomes non-trivial



localization to top layer

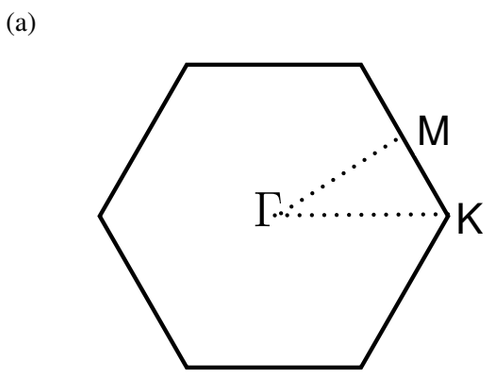
delocalized

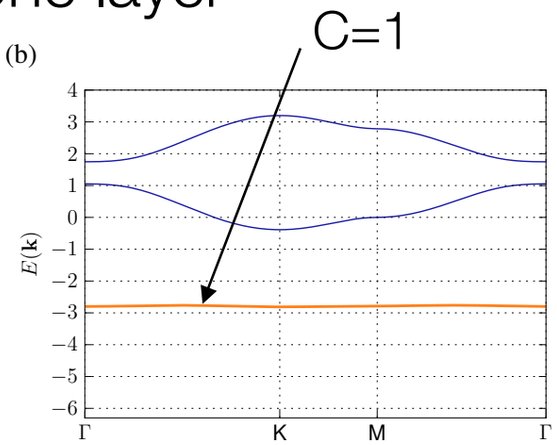
localization to bottom layer

Bulk dispersion and Chern numbers

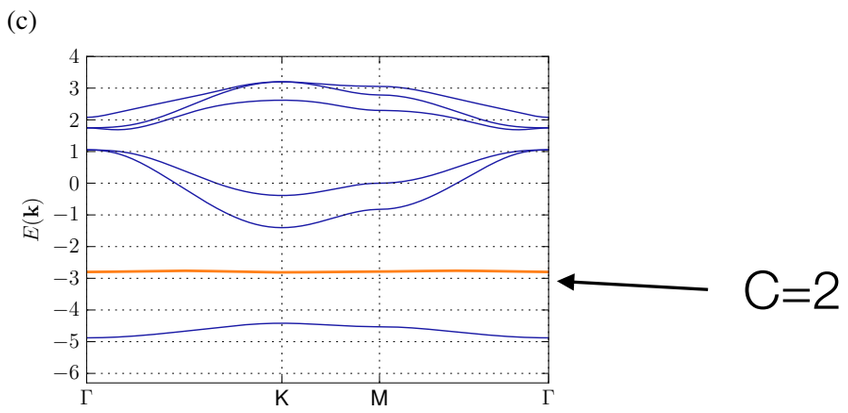
M. Trescher and E.J. Bergholtz,
Phys. Rev. B 86, 241111(R) (2012)

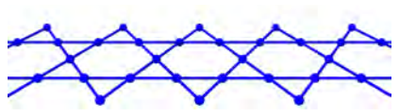
- Dispersion for one layer

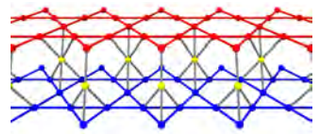




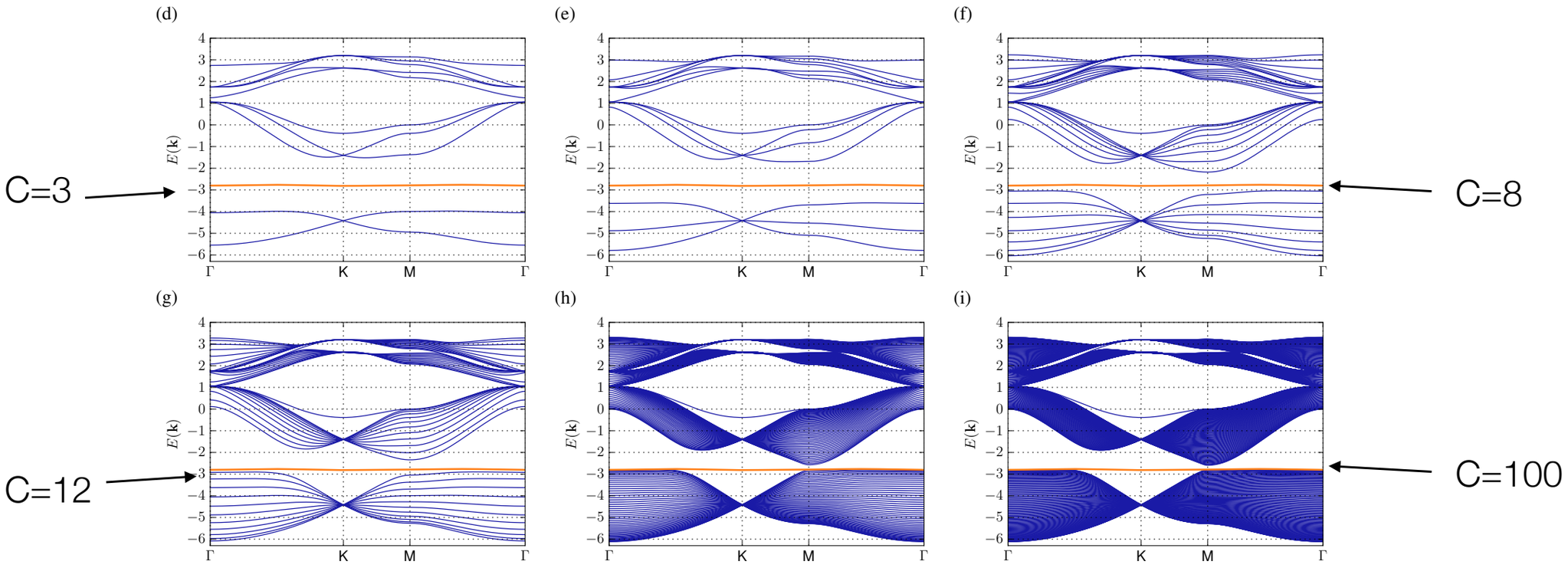
- Dispersion with two layers





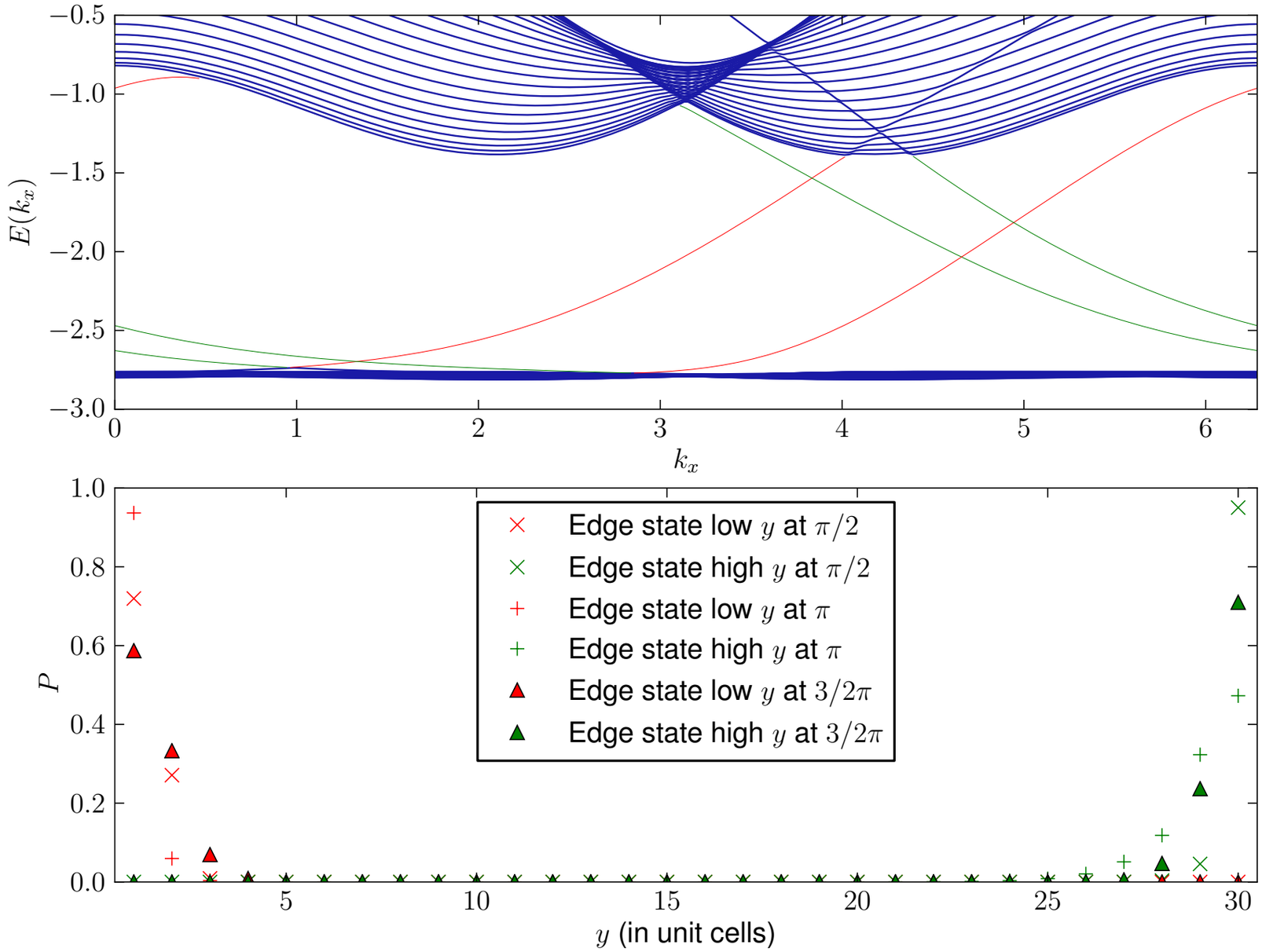


- For N kagome layers we find an almost flat band with C=N!

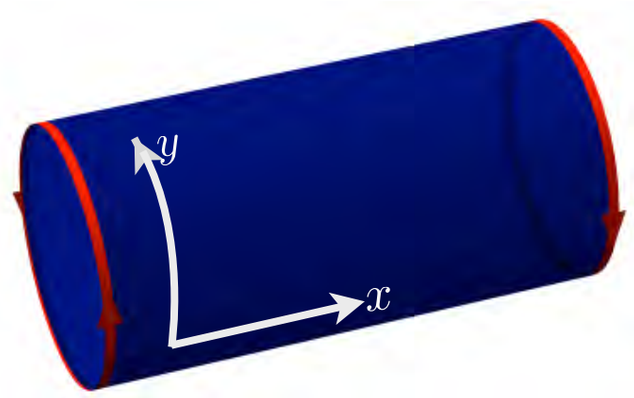


Chiral edge states: revealed in cylinder geometry

- Example: the C=2 band has 2 gapless chiral edge states at each end



(Microscopically different edges to avoid deceiving degeneracies)

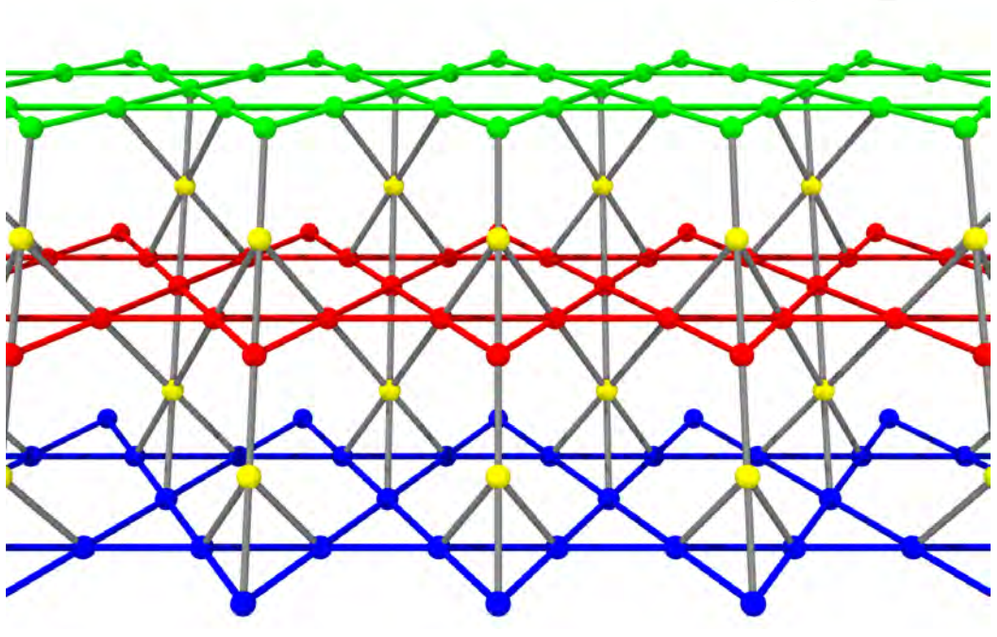


The (almost) flat bands are the surface bands constructed earlier

- N bands, each with C=1, hybridize so that the surface band absorbs all the topology (C=N) while the others become trivial (C=0)

- Simple way of generating (flat) bands with any Chern number

$$|\psi^i(\mathbf{k})\rangle = \mathcal{N}(\mathbf{k}) \sum_{m=1}^{N} \Big(r(\mathbf{k})\Big)^m |\phi^i(\mathbf{k})\rangle_m$$



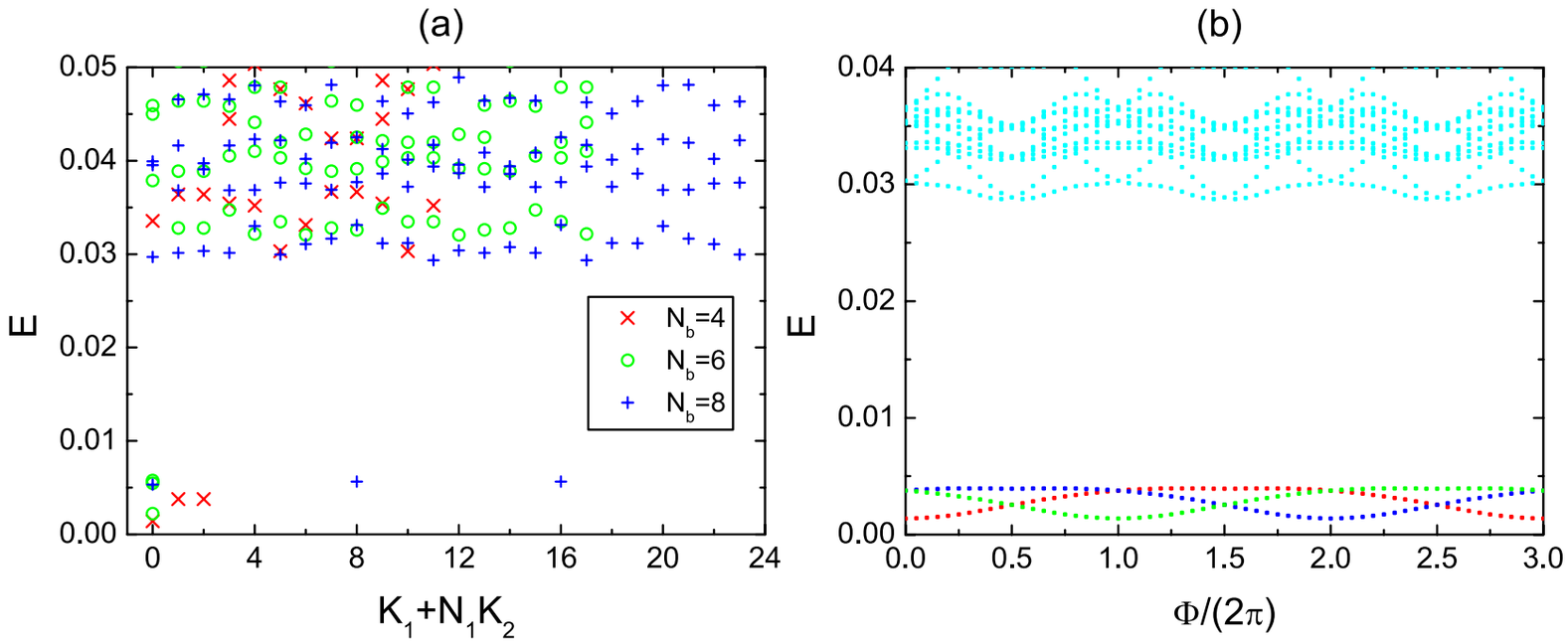
$$\Psi(\mathbf{k}) = \mathcal{N}(\mathbf{k}) \begin{pmatrix} r^2(\mathbf{k})\phi_1(\mathbf{k}) \\ r^2(\mathbf{k})\phi_2(\mathbf{k}) \\ r^2(\mathbf{k})\phi_3(\mathbf{k}) \\ 0 \\ r(\mathbf{k})\phi_1(\mathbf{k}) \\ r(\mathbf{k})\phi_2(\mathbf{k}) \\ r(\mathbf{k})\phi_3(\mathbf{k}) \\ 0 \\ \phi_1(\mathbf{k}) \\ \phi_2(\mathbf{k}) \\ \phi_3(\mathbf{k}) \end{pmatrix}$$

Are local interactions giving new FCI phases within the C>1 bands?

Z. Liu, E.J. Bergholtz, H. Fan, A. M. Läuchli
Phys. Rev. Lett. 109, 186805 (2012)

- Yes, we have convincing evidence for a series of bosonic FCI states at $\nu_b = 1/(C+1)$

$C = 2, \nu_b = 1/3$:



- Fermionic states at $\nu_f = 1/(2C+1)$ (absent at higher filling fractions for local interactions)

- Strong evidence also for C>1 generalizations of non-Abelian FQH states found in this model!

E.J. Bergholtz, Z. Liu, M. Trescher, R. Moessner, and M. Udagawa, Phys. Rev. Lett. 114, 016806 (2015)

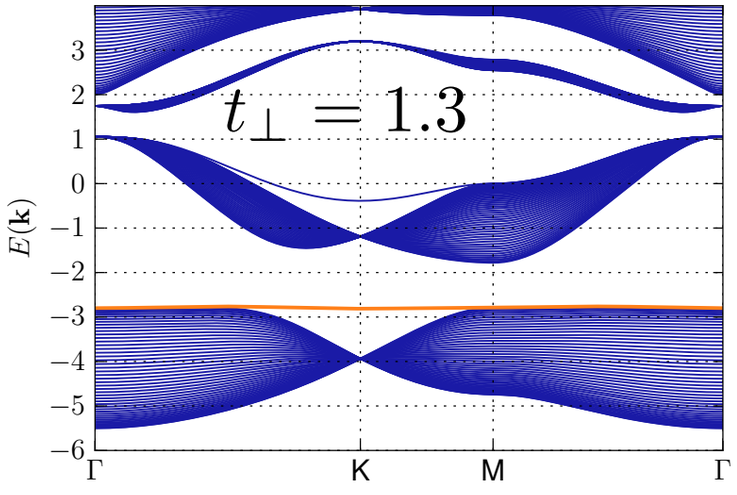
A. Sterdyniak, C. Repellin, B.A. Bernevig, and N. Regnault, Phys. Rev. B 87, 205137 (2013)

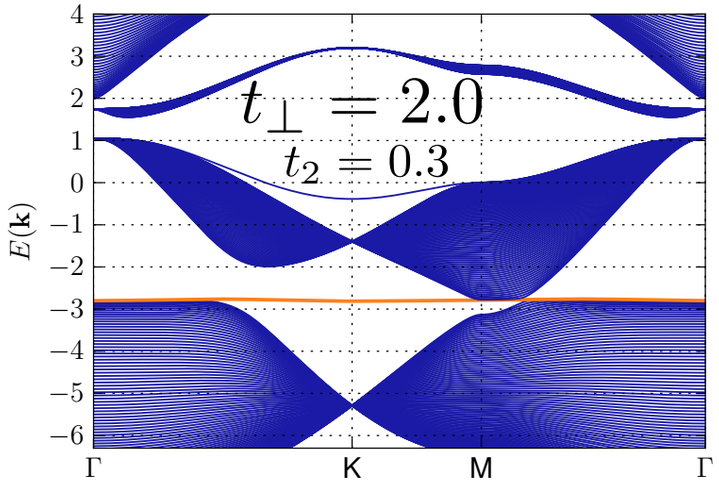
- Different also from standard multi-layer systems (entanglement spectra, boundary conditions, layer-momentum correlation,…)

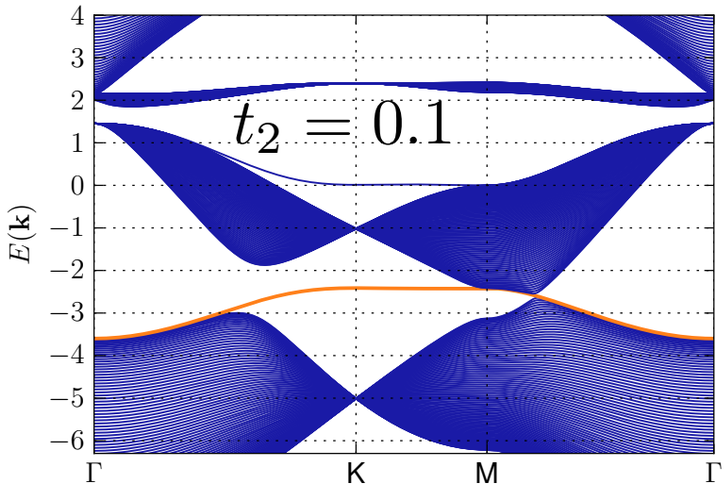
What's the connection to Weyl semimetals?

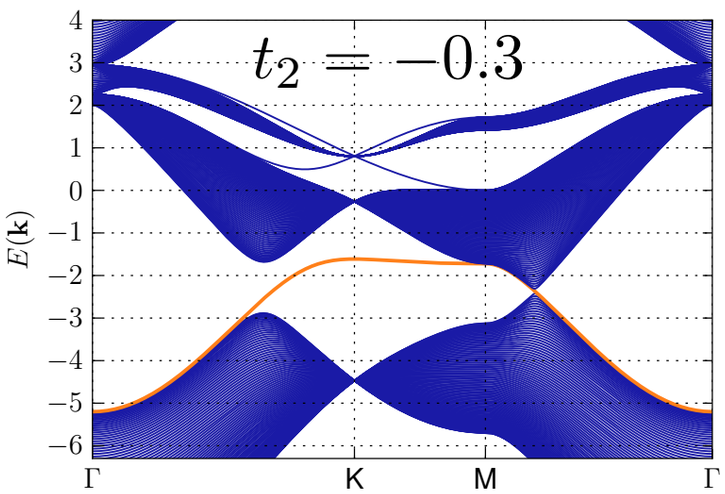
E.J. Bergholtz, Z. Liu, M. Trescher, R. Moessner, and M. Udagawa, Phys. Rev. Lett. 114, 016806 (2015)

- Another look at the bulk spectrum...
- Increase the interlayer tunneling —> bulk phase transition with surface band unchanged!
- Change the nearest neighbor hopping (no change in topology)









- Band touching described by the Weyl Hamiltonian

$$H_{\text{Weyl}} = \sum_{i,j} v_{ij} k_i \sigma_j + E_0(\mathbf{k}) \mathbb{1}$$

  - Nb. this holds in each case, also when the touching cone is nearly flat!

Fermi arcs in the pyrochlore slab

E.J. Bergholtz, Z. Liu, M. Trescher, R. Moessner, and M. Udagawa, Phys. Rev. Lett. 114, 016806 (2015)

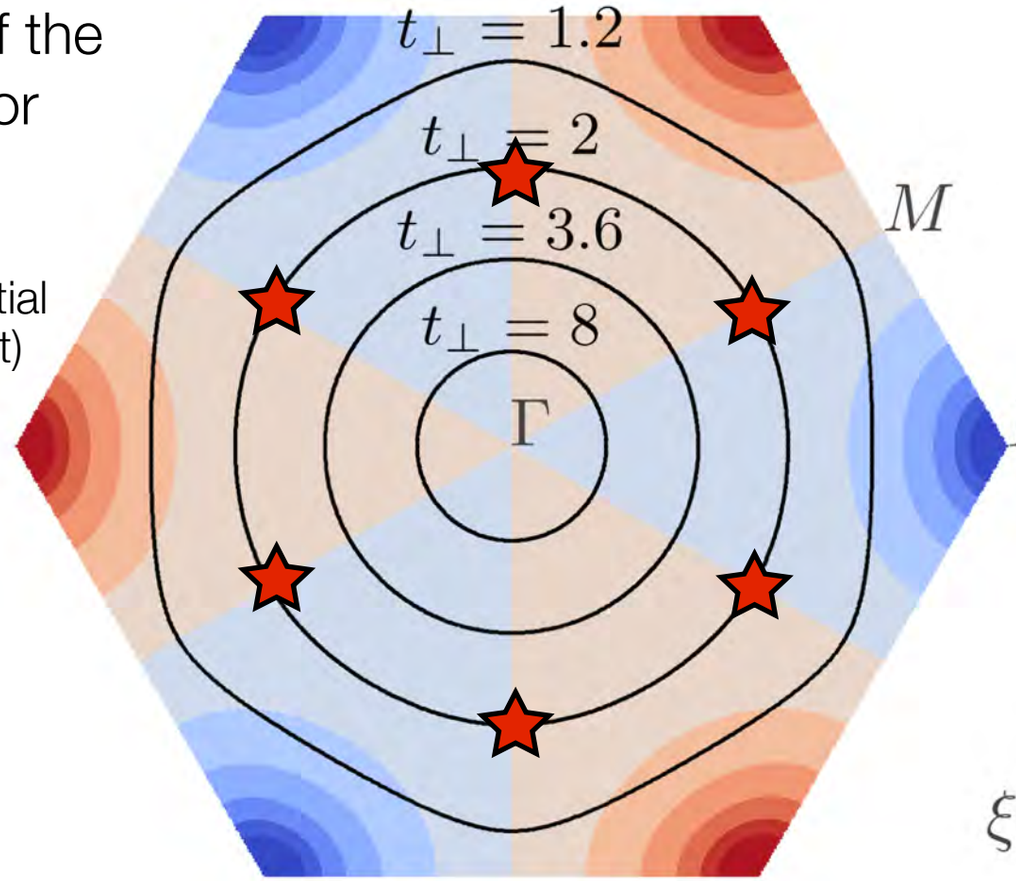
- Constant energy lines, “Fermi circles”, are split into Fermi arcs

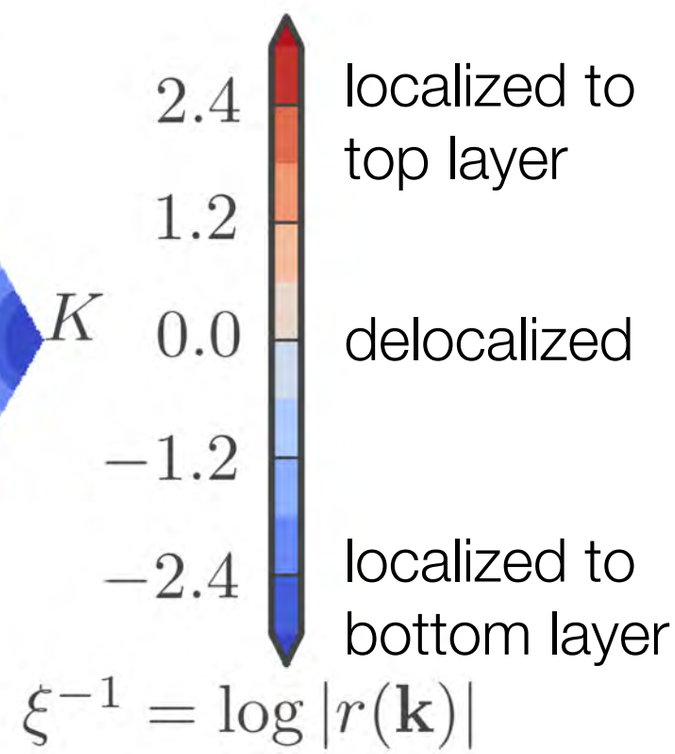
X. Wan, A. M. Turner, A. Vishwanath, and S. Y. Savrasov, Phys. Rev. B 83, 205101 (2011).

★ Projections of the Weyl points for

$$t_\perp = 2$$

(chemical potential at the Weyl point)





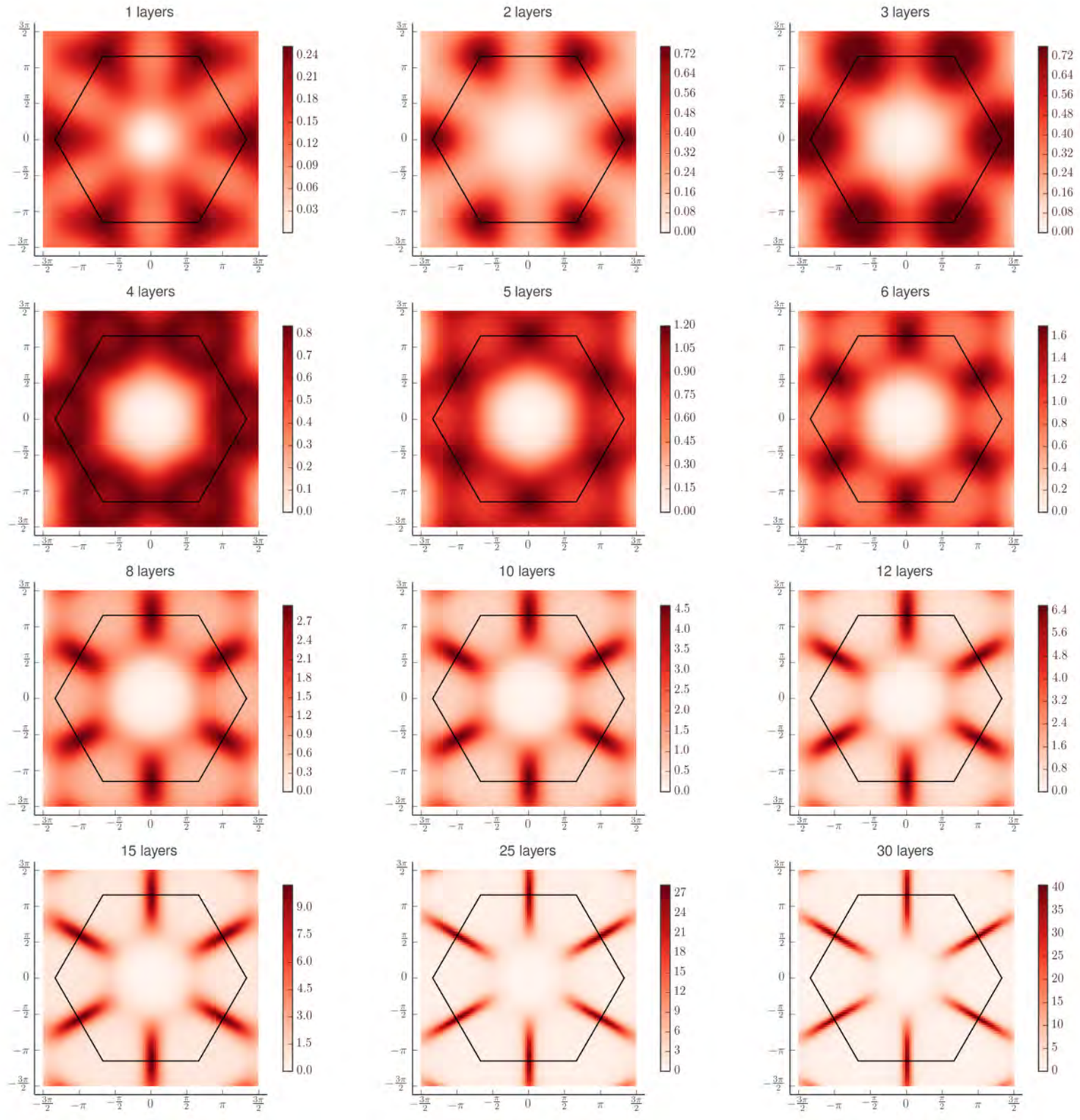
- Here we have an exact solutions for the Fermi arcs, and seen as a family, they carry a huge Chern number.
- The Fermi arcs also exist in absence of Weyl nodes in the bulk!

Berry curvature

- Diverges along lines in the 3D limit

- “topological” property (not dependent on embedding)

- intriguing semi-classical transport



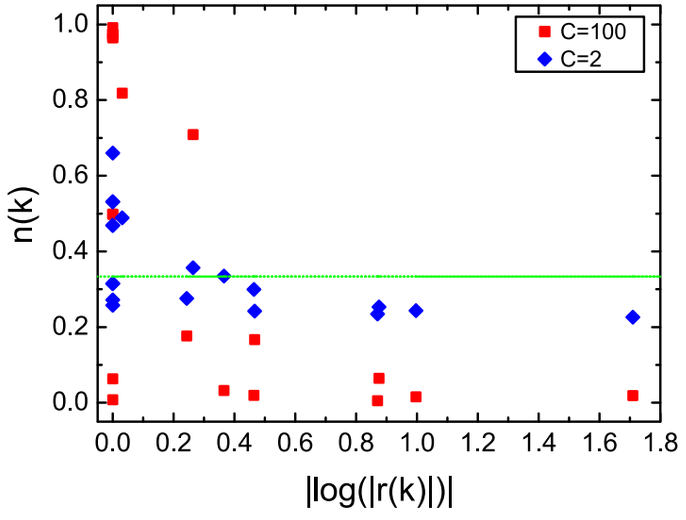
Surface FCIs in the 3D limit!?

E.J. Bergholtz, Z. Liu, M. Trescher, R. Moessner, and M. Udagawa,
Phys. Rev. Lett. 114, 016806 (2015)

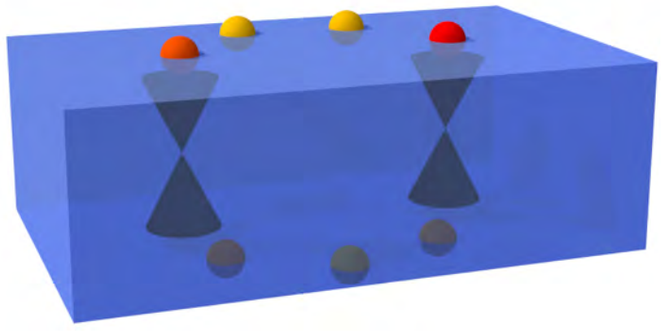
- No!

  - momentum-space density waves compete and eventually prevail

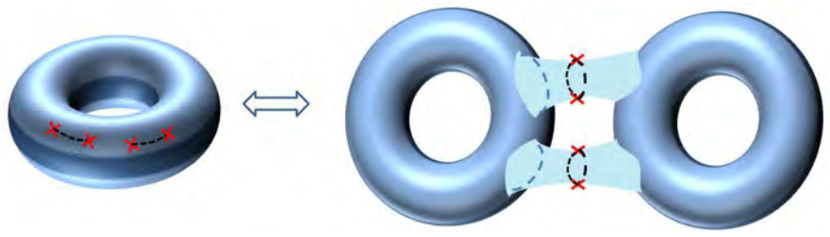
  - due to “leaking” into the bulk along the lines projected onto remnant Weyl nodes (independent of the actual existence Weyl nodes)



- But works fine for finite — even quite thick — slabs



- “Second generation” of fractionalization — phenomenology essentially unexplored



+ ??

M. Barkeshli and X.-L. Qi,
Phys. Rev. X 2, 031013 (2012)

A teaser on

Bulk quantum transport of disordered Weyl semimetals

References:

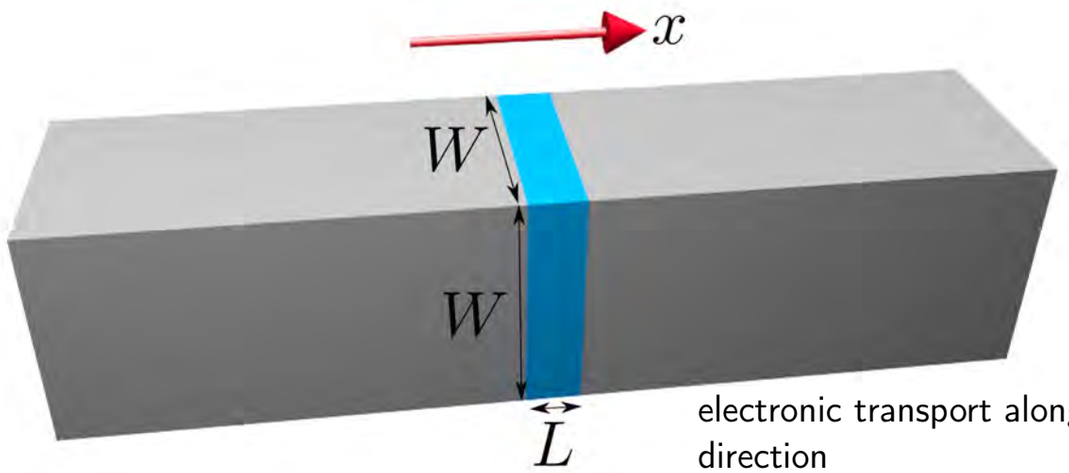
B. Sbierski, G. Pohl, E. J. Bergholtz, and P.W. Brouwer
Quantum transport of disordered Weyl semimetals at the nodal point
Phys. Rev. Lett. 113, 026602 (2014)

M. Trescher, B. Sbierski, P.W. Brouwer, and E. J. Bergholtz
Quantum transport in Dirac materials: signatures of tilted and anisotropic Dirac and Weyl cones
Phys. Rev. B 91, 115135 (2015)

B. Sbierski, E. J. Bergholtz, and P.W. Brouwer
Quantum Critical Exponents for Disordered Three-Dimensional Dirac and Weyl Semimetals
arXiv:1505.07374

Quantum transport

- Setup



electronic transport along x direction
blue: scattering region
grey: leads
$W \gg L$

J. Tworzydło, B. Trauzettel, M. Titov, A. Rycerz, and C.W.J. Beenakker, Phys. Rev. Lett. 96, 246802 (2006)

- Key quantities:

Conductance $G = \frac{e^2}{h}\left(\frac{W}{2\pi}\right)^{d-1} \int d^{d-1}k_\perp T(k_\perp)$

$G = \frac{e^2}{h}\left(\frac{W}{L}\right)^{d-1} g$

g is a dimensionless "cube conductance" (=conductivity only in 2D)

Fano factor (shot noise/current) $F = \frac{\int d^{d-1}k_\perp T(k_\perp)[1-T(k_\perp)]}{\int d^{d-1}k_\perp T(k_\perp)}$

- Clean Weyl cone

- isotropic case:

$$g = \frac{1}{\pi}, \quad F = \frac{1}{3} \quad (d=2),$$

$$g = \frac{\ln 2}{2\pi}, \quad F = \frac{1+2\ln 2}{6\ln 2}, \quad (d=3)$$

- general case:

g finite value, depends sensitively on tilts and anisotropies

$F \geq F_{\text{isotropic}}$ precise value depends only on tilts

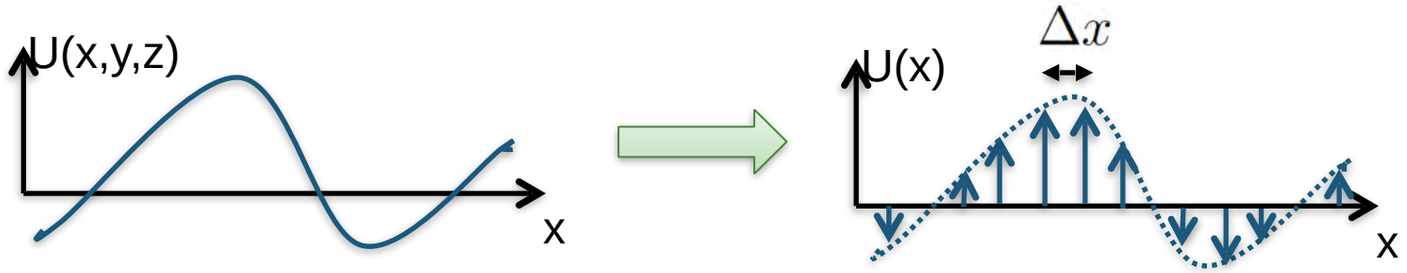
M. Trescher, B. Sbierski, P.W. Brouwer, and E. J. Bergholtz Phys. Rev. B 91, 115135 (2015)

Characterisation of two distinct phases

- Focus: transport properties at the 3D nodal point with disorder

  - known to have two distinct phases — how to characterise them in transport? (Fradkin '86)

- Numerical approach



J. H. Bardarson, J. Tworzydlo, P. W. Brouwer, and C. W. J. Beenakker, Phys. Rev. Lett. 99, 106801 (2007).

free propagation

$$S = S_N^{imp} \otimes S_{\Delta x}^0 \otimes S_{N-1}^{imp} \otimes S_{\Delta x}^0 \otimes \ldots \otimes S_1^{imp} \otimes S_{\Delta x}^0$$

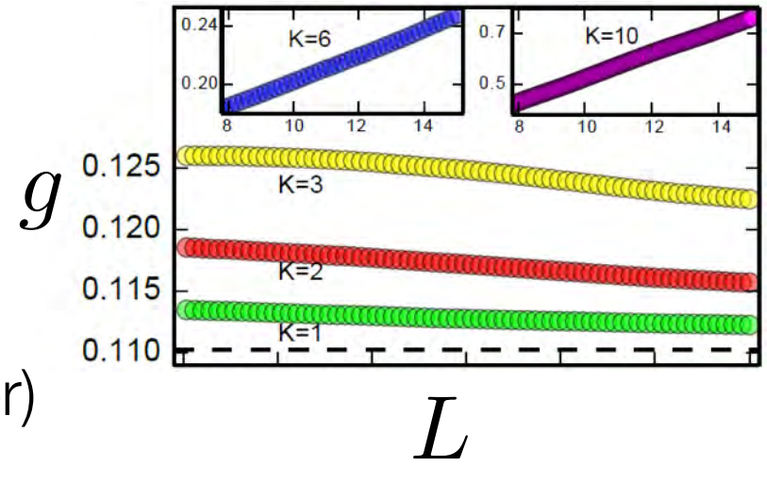
scattering (Born, q–mixing)

- Characterisation of two distinct phases



$$F \approx F_{\text{clean}} \geq 0.57 \qquad F = 1/3$$
$$g \approx g_{\text{clean}} < \infty \qquad g = \sigma L$$

- zero conductivity, finite scale invariant (cube) conductance!



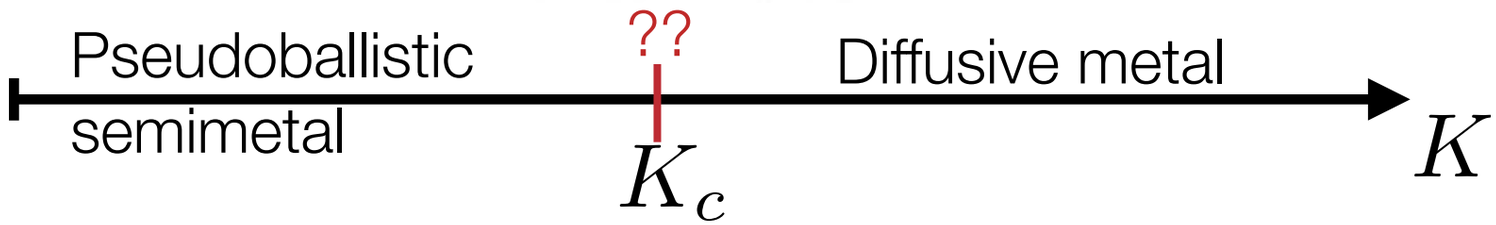
B. Sbierski, G. Pohl, E. J. Bergholtz, and P.W. Brouwer Phys. Rev. Lett. 113, 026602 (2014)

Criticality from finite size scaling of conductance and Fano factor

B. Sbierski, E. J. Bergholtz, and P.W. Brouwer, arXiv:1505.07374

- Quantum criticality described by the two exponents ν and z

$$\zeta \propto |K - K_c|^{-\nu}$$
$$\epsilon \propto \zeta^{-z}$$
$$DOS \propto |K - K_c|^{\nu(3-z)}$$



Crucial for experiments!

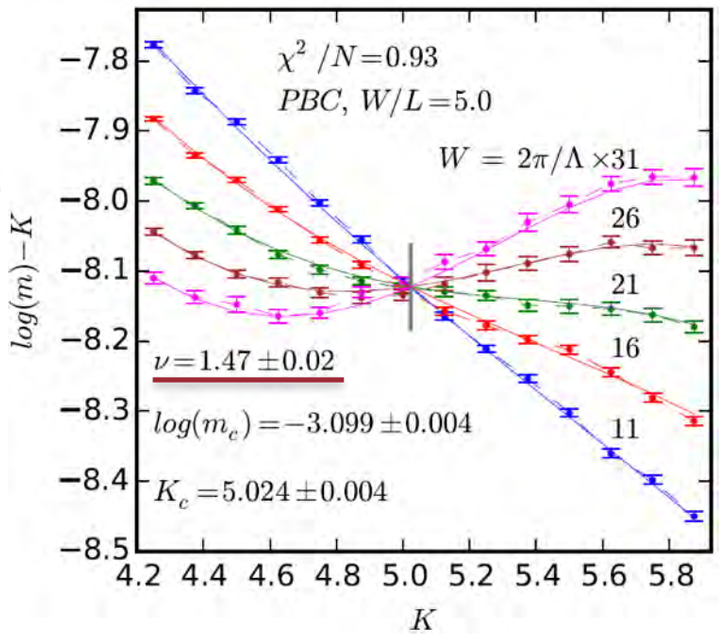
- Diverging results in the literature

Epsilon expansion (one loop): $\nu = 1, z = 1.5$
Epsilon expansion (two loop): $\nu = 1.14, z = 1.31$
Tight binding, DOS scaling: $\nu \approx 0.9, z \approx 1.5$

P. Goswami, and S. Chakravarty, Phys. Rev. Lett. 107,196803 (2011)
B. Roy and S. Das Sarma, Phys. Rev. B 90, 241112(R) (2014)
K. Kobayashi, T. Ohtsuki, K.-I. Imura, and I. F. Herbut, Phys. Rev. Lett. 112, 016402 (2014)

- Key numerical hurdle: determination of K_c
  - Overcome by finite size scaling
- Many disorder realizations, several fixed aspect ratios and boundary conditions
  - Excellent mutual agreement for large enough aspect ratios.
  - Very small error bars — but in sharp contradiction to *all previous studies!*

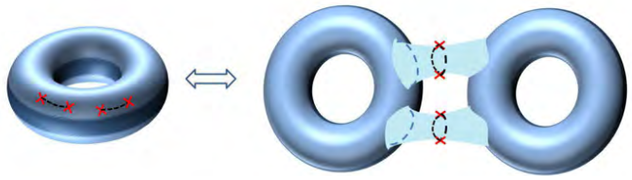
$$\nu = 1.47 \pm 0.03, z = 1.49 \pm 0.02$$

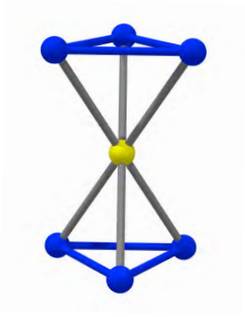


Conclusions, outlook

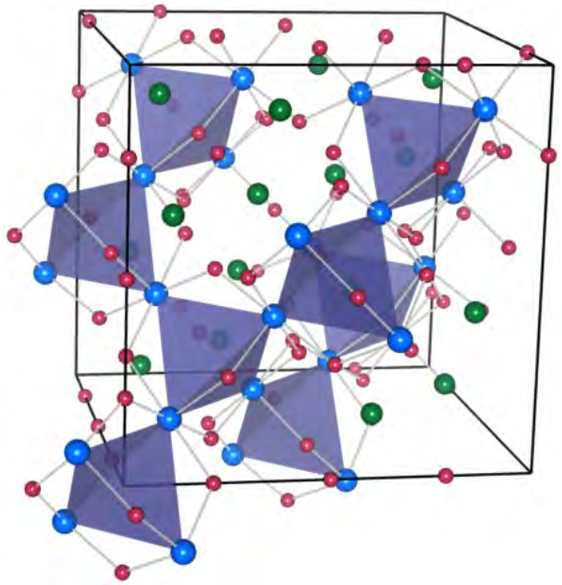
- Frustration adds a novel twist on topology

  - Many open questions and directions!





+ ??



E.J. Bergholtz, Z. Liu, M. Trescher, R. Moessner, and M. Udagawa,
Phys. Rev. Lett. 114, 016806 (2015)

- Disorder induces an intriguing phase transition between two conducting phases in 3D

  - Details controversial, faithful field theory description lacking



B. Sbierski, E. J. Bergholtz, and P.W. Brouwer,
arXiv:1505.07374