On ‘Solving’ a quantum many body problem by experiment

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Quantum fields <-> Correlation functions

✧ Solving a quantum many-body problem is equivalent to knowing all its correlation functions.

✧ In practice, an observer can only measure a finite number of correlations describing the propagation and scattering of excitations.

✧ To solve a problem one need to find degrees of freedom where only few (low order) correlation functions are relevant.

✧ If one finds the degrees of freedom (basis) where the correlation functions factorize, this is equivalent to diagonalization of the many body Hamiltonian.


On the Green’s functions of quantized fields
J. Schwinger PNAS (1951)
1d System

Correlation functions
- fields <-> phase <-> excitations

High order correlation functions
- Quantifying factorization
- Sine-Gordon model
- Quench to a free system

Quantum Field Tomography

Outlook
- entanglement and spin squeezing
- relaxation in SG model

System under investigation

1d - quantum gas
Weakly interacting 1d Bose gas

All energies $\mu, k_B T \ll \hbar \omega$

quasi-condensate

uniform density fluctuating phase

thermally populated

The longitudinal phase fluctuations are key for our experiments

Lieb-Liniger model

- Exactly solvable integrable theory

low energy effective field theory:

**Luttinger-liquid**

$$H = \frac{c}{2} \int dx \left[ \frac{K}{\pi} (\nabla \phi)^2 + \frac{\hbar}{\pi} \frac{\partial}{\partial x} \phi^2 \right]$$

- excitations are soundwaves (phonons)
- linear dispersion relation

coupled 1d systems:

**Sine-Gordon model**

$$\hbar \phi_{(x)} = \frac{\hbar c}{2} \int_{-L/2}^{L/2} dx \left[ \frac{K}{\pi} (\nabla \phi(x))^2 + \frac{\hbar}{\pi} \frac{\partial}{\partial x} \phi(x)^2 \right] - 2m_0 J \int_{-L/2}^{L/2} dx \cos(\sqrt{2} \phi(x))$$

Model for interacting many body systems which can be described by a field theory with long lived excitations.

interference of phase fluctuating 1D condensates

Study the quantum field, its excitations and relaxation

create a copy by splitting quantum connected

create two independent samples classically separated
Evolution after the quench
Decay of the mean contrast

light cone like evolution
Langen et al., Nature Physics 9, 460 (2013)

prethermalized state
Gring et al., Science 337, 1318 (2012)
generalized Gibbs ensemble

slow further decay
thermal equilibrium state

Emergence of classical world from quantum evolution

dephasing of many body eigenstates?
experiments in a trap
-> non translation invariant correlation functions

\[ C(z_1, z_2) = \frac{\langle \Psi_1(z_1) \Psi_2^\dagger(z_2) \rangle}{\langle |\Psi_1(z_1)|^2 \rangle \langle |\Psi_2(z_2)|^2 \rangle} \]

with

\[ \Psi(z) = e^{i\theta(z)} \sqrt{\rho_0(z) + \delta n(z)} \]
\[ \varphi(z) = \theta_1(z) - \theta_2(z) \]

neglecting \( \delta n(z) \)

\[ C(z_1, z_2) \approx \langle \exp[i\varphi(z_1) - i\varphi(z_2)] \rangle \]

4th order:

\[ C(z_1, z_2, z_3, z_4) = \frac{\langle \Psi_1(z_1) \Psi_2^\dagger(z_2) \Psi_3^\dagger(z_3) \Psi_4^\dagger(z_4) \rangle}{\langle |\Psi_1(z_1)|^2 \rangle \langle |\Psi_2(z_2)|^2 \rangle \langle |\Psi_3(z_3)|^2 \rangle \langle |\Psi_4(z_4)|^2 \rangle} \]

\[ C(z_1, z_2, z_3, z_4) \approx \langle \exp[i\varphi(z_1) - i\varphi(z_2) + i\varphi(z_3) - i\varphi(z_4)] \rangle \]
Correlation functions
excitations <-> phase

in experiment we measure the phase $\varphi(z)$
directly

$C^{(2)}(z_1, z_2) = \langle [\varphi(z_1) - \varphi(z_2)]^2 \rangle = \langle [\Delta \varphi(z_1, z_2)]^2 \rangle$

with $\Delta \varphi(z_1, z_2) = \varphi(z_1) - \varphi(z_2)$

Note: $\Delta \varphi$ is NOT restricted to $2\pi$

using

$\varphi(z) = \frac{1}{\sqrt{L}} \sum_{k \neq 0} (-i) \frac{\pi}{|k| L} (b_k^\dagger - b_{-k}) e^{ikz}$

$\langle [\varphi(z_1) - \varphi(z_2)]^2 \rangle = \sum_{k_1, k_2} \frac{\pi}{K \sqrt{|k_1 k_2|}} b_{k_1}^\dagger b_{-k_2} e^{ik_1 z_1 + ik_2 z_2 + \ldots}$

$\Rightarrow$ phase correlators are related to the quasi particles

$4^{th}$ order

$C^{(4)}(z_1, z_2, z_3, z_4) = \langle [\varphi(z_1) - \varphi(z_2)]^2 [\varphi(z_3) - \varphi(z_4)]^2 \rangle$

$\propto b_{k_1}^\dagger b_{k_2}^\dagger b_{-k_3} b_{-k_4} + \ldots$

$\Rightarrow$ quasi particle scattering

When do higher Correlation Functions factorize?
Quantum Sine-Gordon model:

\[ \hat{H}_{SG} = \int dz \left[ \frac{\hbar^2 n_{1D}}{4m} (\partial_z \phi)^2 + g \delta \phi^2 \right] - \int dz \, 2J n_{1D} [1 - \cos \phi] \]

Characteristics:
- Phase coherence length: \( \lambda_T = \frac{2\hbar^2 n_{1D}}{(m k_B T)} \)
- Phase (spin) healing length: \( l_J = \sqrt{\hbar/(4mJ)} \)
- Characteristic parameters: \( q = \lambda_T / l_J \)
experiments probe the phase

-> look at the 'connected part' of the phase correlation function

\[ \langle (\Delta \varphi)^2 \rangle_c = \langle (\Delta \varphi)^2 \rangle \]
\[ \langle (\Delta \varphi)^4 \rangle_c = \langle (\Delta \varphi)^4 \rangle - 3 \langle (\Delta \varphi)^2 \rangle^2 \]
\[ \langle (\Delta \varphi)^6 \rangle_c = \langle (\Delta \varphi)^6 \rangle - 15 \langle (\Delta \varphi)^4 \rangle \langle (\Delta \varphi)^2 \rangle + 30 \langle (\Delta \varphi)^2 \rangle^3 \]
\[ \langle (\Delta \varphi)^8 \rangle_c = \langle (\Delta \varphi)^8 \rangle + 420 \langle (\Delta \varphi)^4 \rangle \langle (\Delta \varphi)^2 \rangle^2 - 630 \langle (\Delta \varphi)^4 \rangle^2 - 35 \langle (\Delta \varphi)^4 \rangle^2 - 28 \langle (\Delta \varphi)^6 \rangle \langle (\Delta \varphi)^2 \rangle^2 = 0 \]

characterized by 'Kurtosis'

\[ \gamma_2 = \frac{\langle (\Delta \varphi)^4 \rangle}{3 \langle (\Delta \varphi)^2 \rangle^2} - 1 \]
\[ \gamma_3 = \frac{\langle (\Delta \varphi)^6 \rangle}{15 \langle (\Delta \varphi)^4 \rangle \langle (\Delta \varphi)^2 \rangle^2 - 30 \langle (\Delta \varphi)^2 \rangle^3} - 1 \]
\[ \gamma_4 = \frac{\langle (\Delta \varphi)^8 \rangle}{630 \langle (\Delta \varphi)^4 \rangle^2 + 35 \langle (\Delta \varphi)^4 \rangle^2 + 28 \langle (\Delta \varphi)^6 \rangle \langle (\Delta \varphi)^2 \rangle^2 - 420 \langle (\Delta \varphi)^4 \rangle \langle (\Delta \varphi)^2 \rangle^2 - 1} = 0 \]

Correlation functions for the fields:

\[ C(z_1, z_2) = \frac{\langle \Psi_1(z_1) \Psi_2^\dagger(z_1) \Psi_1^\dagger(z_2) \Psi_2(z_2) \rangle}{\langle |\Psi_1(z_1)|^2 \rangle \langle |\Psi_2(z_2)|^2 \rangle} \]

\[ C(z_1, z_2) \approx \langle \exp[i\varphi(z_1) - i\varphi(z_2)] \rangle \]

\[ C(z_1, z_2) \text{ contains all orders of connected parts} \]

\[ C(z_1, z_2) = \exp \left[ \sum_{k=1}^{\infty} (-1)^k \frac{(\langle \Delta \varphi \rangle)^{2k}}{(2k)!} \right] \]

for Gaussian fluctuations

\[ C(z_1, z_2) = \exp \left[ -\frac{1}{2} \langle (\Delta \varphi)^2 \rangle \right] \]
Observable and non-gaussian measure

To study factorization of correlation functions we look at

\[ C^{(2)}(z_1, z_2) = \langle [q(z_1) - q(z_2)]^2 \rangle = \langle [\Delta q(z_1, z_2)]^2 \rangle \]
\[ C^{(4)}(z_1, z_2, z_3, z_4) = \langle [q(z_1) - q(z_2)]^2 [q(z_3) - q(z_4)]^2 \rangle = \langle [\Delta q(z_1, z_2)]^2 [\Delta q(z_3, z_4)]^2 \rangle, \]

\[ \Delta q(z_1, z_2) = q(z_1) - q(z_2) \]

\[ \Delta q \text{ is NOT restricted to } 2\pi \]

Characterising non-Gaussian phase fluctuations

Characterising the factorisation by the connected part: \( \langle (\Delta \phi)^4 \rangle_c = \langle (\Delta \phi)^4 \rangle - 3 \langle (\Delta \phi)^2 \rangle^2 \)

Excess Kurtosis

\[ \gamma_2 = \frac{\langle (\Delta \phi)^4 \rangle}{3 \langle (\Delta \phi)^2 \rangle^2} - 1 \]

Experimental data, thermal state in a double well

Plasma freq. = 0 Hz
\( q = 0 \)

70 Hz
\( q = 2.5 \)

160 Hz
\( q = 3.4 \)

220 Hz
\( q = 8.6 \)
the breakdown of factorization is evident in the full distribution functions of the phase by new peaks at multiples of $2\pi$

caused by the $2\pi$ periodic SG Hamiltonian -> $2\pi$ phase jumps, 'kinks' = SG solitons

SG Solitons are topological excitations

Phase fluctuations around topologically different Vacua
6-point phase correlators

<table>
<thead>
<tr>
<th>$q$</th>
<th>0</th>
<th>2.5</th>
<th>3.4</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>$z_1$ (μm)</td>
<td>0</td>
<td>20</td>
<td>0</td>
<td>20</td>
</tr>
<tr>
<td>$z_2$ (μm)</td>
<td>0</td>
<td>20</td>
<td>0</td>
<td>20</td>
</tr>
</tbody>
</table>

full
Wick factorization
difference
lower limit
upper limit

6-point phase correlators, connected part

<table>
<thead>
<tr>
<th>$\omega_p$</th>
<th>0 Hz</th>
<th>70 Hz</th>
<th>160 Hz</th>
<th>big</th>
</tr>
</thead>
<tbody>
<tr>
<td>$z_1$ (μm)</td>
<td>0</td>
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<td>0</td>
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<td>20</td>
</tr>
</tbody>
</table>

full
disconnected part
connected part
lower limit
upper limit
Remove Solitons
Strongly coupled \( q = 8.6 \quad \omega_p > 500 \text{ Hz} \)

4-point correlator does not factorize:

without Solitons:

phase distribution:

different sectors:

Remove Solitons
intermediate coupling \( q = 3.4 \quad \omega_p = 160 \text{ Hz} \)

4-point correlator does not factorize:

without Solitons:

phase distribution:

without solitons:
What have we learned

- **high order (>10) correlation** functions are accessible in experiment
- **full distribution functions** and the **connected part** of the higher order correlation functions contain genuine information about the quantum field theory
  - quasi particles
  - interaction of quasi particles
  - vacuum states
- gives insight in the **effective theories** describing the many body system
  - for our resolution 6th order is sufficient
  - → necessary to take perturbation expansion up to 3rd order (3-3 scattering)

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Quench from J>0 to J=0

Initial state q=3,4: non-Gaussian, dynamics Gaussian

Collaboration with Berges & Gasenzer groups, Heidelberg
Quench from $J>0$ to $J=0$

Initial state $q=3,4$: non-Gaussian, dynamics Gaussian

Experiment: Steffens et al. nature communications (2015) arxiv:1406.3632

www.AtomChip.org
Reconstructing Quantum States

- Reconstruction of an unknown state based on data alone
- Generically, need $d^2$ expectation values to reconstruct an unknown state in $d$ dimensions
- full tomography tools for state identification inefficient, especially for continuous systems
- Brought down to $O(rd \log^2(d))$ for approx low-rank state with compressed sensing.
- Applicable for medium sized systems in conjunction with model selection. Approaches are based on using the right “data set” with the appropriate “sparsity structure” to capture quantum many-body systems.
  - over permutation-invariant tomography
  - matrix-product state tomography
  - ....

Quantum Field Tomography

efficient tomographic state reconstruction for continuous quantum fields

continuous Matrix Product States (cMPS) naturally incorporate the locality present in realistic physical settings of locally interacting quantum field

Phase correlation functions

$$C^{(n)}(x_1, \ldots, x_n) = \text{Re} \left\langle e^{i(\hat{x}_{x_1} - \hat{x}_{x_2} + \hat{x}_{x_3} - \ldots - \hat{x}_{x_{n-1}} - \hat{x}_{x_n})} \right\rangle$$

with

$$\psi^+(x) = \hat{n}(x)^{1/2} e^{\hat{x}}$$

Extract low-order correlation functions

$$C^{(n)}(\tau_1, \ldots, \tau_{n-1}) = \sum_{\{k_j\} \geq 1} \rho_{k_1, \ldots, k_{n-1}} e^{\lambda_{k_1}} \ldots e^{\lambda_{k_{n-1}}} \tau_{n-1}$$

Reconstruct continuous matrix product states

$$|\psi_{Q,R}\rangle = \text{tr}_{aux}(\mathcal{P} e^{\int_0^L dx (Q \otimes 1 + R \otimes \psi^+(x))} |0\rangle$$

Methods: Matrix pencils, prony methods

**Quantum field tomography**


Use 2- and 4-point functions to reconstruct higher-order functions via continuous matrix product states with bond length 2

reconstruction of a quantum field with very weak assumptions

**Theory:**

J. Schmiedmayer: On 'Solving' a Quantum Many-Body Problem by Experiment

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**Quantum field tomography**


Use 2- and 4-point functions to reconstruct higher-order functions via continuous matrix product states with bond length 2

reconstruction of the C-MPS wave functions gets worse with time
C-MPS with bond length 2 have finite entanglement
Question: Can one build a measure for entanglement growth after the quench?

similar to data compressibility criteria?
Outlook

Non trivial (squeezed) initial states
Relaxation in SG moel

Optimal Control of Splitting
fast squeezing in a multi mode system

Optimal Control applied to the problem of the fluctuation properties in splitting a BEC
J. Grond et al. PRA 79, 021603 R (2009)
J. Grond et al. PRA 80, 053625 (2009)

- Fancy splitting ramps inspired by OCT: $t_1 + t_2 = 17$ ms
- Leads to dramatic change of statistical distribution of interference

full cloud 140 µm long
J. Schmiedmayer: On ‘Solving’ a Quantum Many-Body Problem by Experiment

\[ \Delta n \Delta \phi = 2.3 \pm 0.7 \]

when correcting for measurement noise: \( \Delta n \Delta \phi \sim 1 \)

number and phase distribution

Squeezing RMS fluctuations of the number difference

Whereas RMS fluctuations of the phase

\[ \approx 150 \text{ atoms} \]

Spin squeezing:


Evolution of \( \xi^2 \sim -8 \text{dB} \) 1d gas

Tunnel Coupled \( \omega_p=14 \text{Hz} \)

Separated

T. Berrada preliminary
Relaxation in coupled superfluids

re-coupling starts SG model with a specific phase

\[ \rightarrow \text{study phase locking} \]

\[ \text{phase locking as a fix-point of the evolution} \]

What have we learned

- high order (>10) correlation functions are accessible in experiment
- Higher order correlation functions and the question if they factorize (full distribution functions) gives insight in the effective theories describing the many body system
  - quasi particles
  - interaction of quasi particles
  - vacuum states
- Quantum field tomography opens up a way to extract information by using model cMPS wave-functions
- Experiments allow to probe how classical statistical properties emerge from microscopic quantum evolution through de-phasing of many body eigenstates.

Schweigler et al. arXiv:1505.03126
Steffens, et al., NJP 16, 123010 (2014)
**Papers**

Probing Quantum Fields by correlations  
Schweigler et al. arXiv:1505.03126

Quantum Field Tomography  
Steffens, et al., NJP 16, 123010 (2014) (theory)

Non equilibrium an relaxation in 1d systems  
Gring et al., Science 337, 1318 (2012)  
Kuhnert et al., PRL 110, 090405 (2013)  
Smith et al. NJP 15, 075011 (2013)  
Langen et al., Nature Physics 9, 460 (2013)  
Geiger et al. NJP 16 053034 (2014)  

Interferometer with trapped BEC  

Coolig a 1d quantum gas  
Rauer et al., arXiv:1505.04747

**Atom Chip Experiment**

S. Manz, T. Betz, R. Bucker, T. Berrada, S. vanFrank, M. Pigeur, A. Perrin, T Schumm, JF Schaff, R. Wu, M. Bonneau  
M. Kuhnert, M. Gring, B. Rauer, Th. Schweigler  
D. Smith, R. Geiger, T. Langen

**Atom Chip Fabrication**  
D. Fischer, M. Trinker, M. Schamböck (ATI)  
S. Groth (HB), Israel Bar Joseph (WIS)

**Theory Collaboration**  
I. Mazets, P. Grisins (ATI)  
J. Grond, U. Hohenester (Univ. Graz)  
E. Demler, T. Kitagawa + ... (Harvard)  
T. Gasenzer, J. Berges, S. Erne, V. Kasper + ... (Heidelberg)  
T. Calarco, S. Montangero + ... (Univ. Ulm)  
J. Eisert + ... (FU-Berlin)

**EU:** SIQS, QIBEC, AQUS  
**AT:** FWF, CoQuS, Wittgenstein, Stadt Wien  
**ERC AdG:** QuantumRelax

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